The greatest meeting of the year for all teachers of science and mathematics—The Central Association of Science and Mathematics Teachers—November 26 and 27, Edgewater Beach Hotel, Chicago.

CONTENTS FOR OCTOBER, 1949

THE FORTY-NINTH ANNUAL CONVENTION

November 25 and 26 will witness the convening of the Central Association of Science and Mathematics members at Edgewater Beach Hotel, Chicago. This forty-ninth convention is an important guidepost in the history of Central Association. It marks the advent of a half-century of history-making events in Central Association. Plans are already in progress for the 1950 anniversary; announcements of these will be made at the 1949 sessions.

The general program on Friday, November 25 is slightly modified in order that four sectional programs may be held in the morning, and four in the afternoon. This change will enable members to attend more sections of their choice. One-hour general sessions will be held on Friday morning and Friday afternoon.

Speakers on the Friday general programs will be Dr. Paul C. Aebersold, Chief, Isotopes Division, United States Atomic Energy Commission, Mr. Ronald M. Foster, Chairman, Department of Mathematics, Polytechnic Institute of Brooklyn, New York, and Dr. B. L. Dodds, Professor of Education, Purdue University. The Saturday morning general session will be addressed by Dr. C. W. Sanford, Director, Illinois Secondary School Curriculum Program and Professor of Education, University of Illinois, Urbana.

The banquet on Friday evening will be presided over by hosts and hostesses. The speaker of the evening will be Dr. C. C. Furnas, whom so many of you know because of his books. He is now Executive Vice-president and Director of the Cornell Aeronautical Laboratory at Buffalo, New York. His address carries the thought-provoking title, *Civilization in a Quandary.* An Association mixer will follow the banquet, designed for you to meet your friends and get acquainted with Association speakers and officers.

The annual trip on Saturday afternoon will be made this time to the Institute of Nuclear Studies, University of Chicago. Dr. Samuel

K. Allison, world-famed scientist, will act as host and explain the work now being carried out in the field of atomic energy.

The 1949 Yearbook will carry the entire program of the Central Association Convention. Sectional and group topics are timely and adapted to the needs and interests of teachers.

Hotel reservations should be made as soon as possible with Mr. E. J. Ahern, Reservation Manager, Edgewater Beach Hotel, Chicago. Advance reservations for the banquet on Friday and trip on Saturday afternoon may be made with Mr. Milton D. Oestreicher, Coördinator of Local Arrangements, Francis W. Parker School, 330 W. Webster, Chicago.

CHARLOTTE L. GRANT, *President*

MEET THE AUTHORS!

"Pick an editor to be an editor" was possibly the thought which prompted the committee to select Walter N. Carnahan, editor of the mathematics department of D. C. Heath and Company, as editor-in-chief of the anniversary publication of the central Association of Science and Mathematics Teachers. The book, "Fifty Years in Science and Mathematics Education" will make its appearance in 1950 as part of the 50 year celebration of that organization.

Walter N. Carnahan attended Indiana State Teachers College and received his A. B. degree from Oakland City College. He has an M.A. from Indiana University and has done graduate work at the University of Chicago. He served at one time as principal of the high school at Bloomington, Indiana, going from there to Shortridge High School at Indianapolis where he was head of the mathematics department for sixteen years. He was next associated with Purdue University as Mathematics Counsellor and Professor of Education. For the past two years he has been editor of the mathematics department of D. C. Heath and Company and is now associated both with Purdue and that publishing house.

Mr. Carnahan is a past president of the Central Association and has served two terms on the Board of Directors. His family includes Mrs. Carnahan, five children and six grandchildren.

Glen W. Warner hardly needs an introduction to readers of SCHOOL SCIENCE AND MATHEMATICS, of which he is the able editor. He will contribute to the anniversary publication, something concerning the history of this journal.

Dr. Warner received both his A.B. and Ph.D. degrees in Physics from Indiana University and did his other graduate work at the University of Chicago. He taught in the high school at Goshen, Indiana

and was principal of the high school at Globe, Arizona before he went to Chicago to teach, first in the high school at Englewood, and then at Crane Junior College and the Wilson Branch, Chicago Junior College. This year he is retiring from teaching but will continue as editor of SCHOOL SCIENCE AND MATHEMATICS.

Dr. Warner has been a member of the Central Association of Science and Mathematics Teachers since 1913 and has served the association in many capacities. Mrs. Warner is also a familiar figure at the annual conventions of that organization. Son, Lowell C. Warner, followed in his father's footsteps and teaches Physics at Chicago Junior College, Wilson Branch.

Jerome Isenbarger is senior author for the biological section of the anniversary publication. Assisting him is John C. Mayfield. Mr. Mayfield received his B.S. degree from Franklin College and his M.A. from the University of Chicago. He taught in Indiana at Beech Grove and Whiting before going to the University of Chicago High School in 1926. Since 1939 he has been an instructor in biological sciences in The College of the University of Chicago.

Mr. Mayfield is co-author with W. L. Beauchamp and Joe Young West of *Science Problems*, and *Everyday Problems in Science*, and co-author with Dr. Beauchamp of *Basic Electricity*.

Joining the Central Association of Science and Mathematics Teachers the year he received his B.S. degree, Mr. Mayfield has served as a member of the Board of Directors and as a section chairman. The Mayfields have two sons, Arthur and Eugene.

Another junior author is Allen F. Meyer of Detroit, Michigan. He will assist Ira C. Davis with the section on physical sciences.

Mr. Meyer holds an A.B. and an M.A. degree from the University of Michigan and is completing work for a Ph.D. degree from that institution. He has done graduate work at Wayne University, Lawrence Institute of Technology, University of the Pacific, and the University of California at Berkeley. He has taught in the schools at Dearborn, Michigan, and Lodi, California, and is now head of the science department and principal of the evening school at Mackenzie High School in Detroit, Michigan. He is a former editor of the *Metropolitan Detroit Science Review*.

Mr. Meyer has been active in the Central Association of Science and Mathematics Teachers for the past ten years, serving on the Board of Directors, as a section chairman, and a convention manager. Mr. and Mrs. Meyer have a daughter, Anne.

We will introduce you to the other authors of the anniversary publication in the November issue of this journal.

MARIE S. WILCOX

CULTIVATING RESEARCH AND AESTHETIC ATTITUDES IN ELEMENTARY MATHEMATICS

PETER DROHAN

Hartsmere, Ontario, Canada

Nothing encourages an enthusiastic student of mathematics like seeing discoveries he can make with his growing knowledge of the subject. What a feeling of satisfaction and conquest, for instance, a comparative beginner in algebra experiences on realizing that with the help of a few suggestions he is able to demonstrate such a relation as the summation of an arithmetical progression! Just imagine: in a few seconds to be able to write the sum of any number of terms of the series: 1, 2, 3, 4, . . . , n. It savours almost of magic and is a stimulative urge to further effort.

Curiosity is stirred and wonder grows as to what the next thrill may be. Then, unexpectedly, some day the instructor hints a certain manipulation of symbols and operations, and out pops another beautiful principle, that of summing a geometric series. The door is opened to the mathematics of finance and the numerous intriguing problems connected therewith. The mystery surrounding the conversion of a recurring decimal to a vulgar fraction disappears. The twice-told tale of the marvellous cost of shoeing a horse at two cents for the first nail and doubling the charge for each additional nail becomes a reasonable relation of numbers.

New opportunities for discovery will present themselves as the type forms $(a+b)^2$ and $(a+b)^3$ are considered. If both sides of the latter identity are multiplied by $(a+b)$ and the operation repeated two or three times, there will be indications of something unusual. The number of terms in the expansions, the index and coefficient relations, and the regular ascent and descent of the powers of the letters involved cannot fail to challenge the attention of the budding mathematician and lead him to a pinnacle whence he may obtain a glimpse of one of Newton's great discoveries—the binomial theorem. Truly, mathematics is a realm where wonderful relations abound.

However, no greater sense of power can be handed out to the diligent learner who has toiled through the comparatively dry and uninteresting theory of indices than to introduce him to the possibilities it opens up to him. It is the pathway to logarithms that wonderful device which took the arithmetical drudgery out of mathematics and redoubled the speed of calculation. A few properly chosen examples using ten as a base will give the clue to the principles involved and lead the student to see the immense utility of the new artifice for numerical computation. For more advanced pupils who are fully ac-

quainted with the binomial theorem, the suggestion to expand and compute the numerical value of $(1+n)^{1/n}$ when $n \to 0$, brings to light the Napierian base, and a further manipulation of the form $(1+n)^{x/n}$ when $n \to 0$, uncovers another result of far-reaching importance—the exponential theorem.

From the process of measuring the circumference of a circle the way is open to a unit of angular reckoning—the radian. The simple but elegant formula $S/R = \theta$ has a great variety of applications in astronomy, geography and mechanics. It is a fitting companion for pi and the Napierian base.

Synthetic geometry, too, abounds in relations that enthuse the amateur investigator. The beginner who can bisect an angle, draw a perpendicular or construct a simple figure may experimentally discover the nine point circle, the Simpson line, the theorem of Ptolemy, etc. Those who are farther advanced may get a thrill from pursuing the ideas of continuity, the properties of regular polyhedra, and various other conceptions extending into broader and more recondite parts of the subject.

The tangent equation of analytical geometry sets the student on the threshold of really modern mathematics. He is on the road traversed by Liebnitz when he discovered the infinitesimal calculus, the idea which revolutionized mathematical science. The step from the elementary concept of tangent to that of the differential is natural enough, and any high school senior who is keenly interested in mathematics will be delighted with a simple presentation of the great discovery. Of course Newton, a contemporary discoverer of the same principle would have to be considered too. The double birth, so to speak, of the differential, that powerful tool of mathematical analysis would have to be discussed in its relation to the two great original minds who first introduced it bringing the unknown into the domain of the known, the fruit of all intellectual effort.

The cutting of a cone at various angles with the axis is another revelation of the beauties and almost cosmic significance of geometrical lines. Not only are there marvelous relationships of loci, but the lines determined are the paths of the material particles moving in a gravitational field. Devoid of aesthetic appreciation he must be who is not thrilled by this blending of geometry and dynamics. How an instructor is delighted to guide a student through a region so fascinating to a reflective mind. There is such an opportunity to cultivate a learner's attitude towards his subject and develop an urge to find out new knowledge relative to it.

Something else that appeals to a student's curiosity and at the same time introduces him to geometrical conceptions somewhat different from those peculiar to Euclidean space is the suggestion to

consider the properties of figures confined to the surface of a sphere, let us say. The straight lines of the plane are transformed into great circles, and, although they divide the surface into two congruent parts, these parts do not extend to infinity. Thus there would be no infinity or theory of parallels. Some of the analogies and differences of plane geometry and geometry of a spheric surface are quite manifest and the references, often heard, to non-Euclidean space become quite significant. An account of Euclid's parallel axiom is in order here and the conclusions to which it led: no parallel, one parallel, or an infinity of parallels, may be illustrated by diagrams.

In the preceding paragraphs a few points have been given whereby students might be led to interesting discoveries far in advance of where they are working in the ordinary textbooks, or given glints of outstanding developments in mathematics with a view to creating enthusiasm, or mayhap fostering a spirit of investigation destined to bear fruit some time in the future. The aesthetic element of mathematics, too, might be furthered by an emphasis on the harmonious numerical and spacial relations so constantly in evidence. Nowhere else can be found such a stimulus to precision of statement, such a wealth of conceptual beauty, and such a sweep of intellectual power; the responsibility rests on the instructor to develop these aspects, thus nourishing the spirit in an atmosphere of truth, beauty, and breadth of view.

THE QUIZ SECTION

Julius Sumner Miller
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1. A projectile strikes a target at 45° and rebounds at the same angle. Show that $\mu = 1 - e/1 + e$.
2. A projectile is fired and hits a target distance d away at right angles. Show that it will fall a distance de (d times e) from the target.
3. Two billiard balls stand in contact. A third strikes the pair simultaneously, and is instantly brought to rest. Find e.
4. A particle is shot from a point on a smooth horizontal plane with velocity V at an angle of elevation A. It strikes the plane and rebounds time after time. Find the total time of flight and the total range. Find also the total path traversed.
5. A hammer of mass M falls from a height h and hits an inelastic pile of mass m which is consequently driven down a distance d. Find the time during which hammer and pile are moving together.

PACIFIC INDUSTRIAL CONFERENCES

"Twenty thousand chemists, chemical engineers, and industrial executives from all over the nation and many other countries are expected to attend the Pacific Industrial Conferences which will run concurrently with the Pacific Chemical Exposition at the San Francisco Civic Auditorium, November 1–5, 1949," says Dr. Richard Wistar, chairman of the California Section of the American Chemical Society.

SOME ATOMIC PARTICLES AND THE MESON

JAMES I. SHANNON

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1. The electron has passed the half-century mark in its history as a known particle. It forms a part of every single atom of matter of what kind soever. Its properties have been thoroughly canvassed and it has been put to surprising tasks. For the electron has a number of very comfortable qualities: (1) Its charge is always of the same magnitude whatever may be its origin. It is therefore an extremely useful unit for measuring all the charges which we meet in the commerce of the atomic world; (2) It is extremely light and therefore can be readily made to move at speeds that are dizzying to our imagination; (3) It can be readily deflected and made to move in a different direction.

2. Another very surprising result of atomic research was the establishment of the nucleus as a tiny particle at the center of the atom, its diameter being only about one ten-thousandth of that of the atom. Yet this tiny speck contains all the positive electricity of the atom and practically all of its mass. For outside the nucleus are only some electrons, in number never exceeding 96, and having between them an extremely small fraction of the mass of the nucleus. This fraction varies from about 1/1840 in the case of the hydrogen atom to about 1/20000 in the case of uranium. The nuclear theory was proposed by Sir Ernest Rutherford and established by him during the second decade of this century.[1] The atom according to this view is a sort of miniature solar system with the nucleus as the sun and the electrons as the planets which circulate about the nucleus and are held to their courses by a nice balance between the centrifugal tendency of the electrons and the mutual attraction between them and the nucleus. The analogy is very useful but should not be pushed too far, for while the planets attract each other, the electrons repel other electrons.

3. This tiny nucleus might seem the ultimate in diminishing size, for it is hopelessly beyond the power of our best microscopes to body forth. Yet the nucleus is not an indivisible entity. It has parts:

(1) Alpha rays or alpha particles are flung out by the atoms of certain elements, and they unquestionably come from the nucleus. They bear a positive charge which is of exactly two units, that is twice the charge on an electron.

(2) Beta particles, very minute, but still particles of a definite mass are thrown out by the atoms of a number of substances under circumstances that show they come from the nucleus. The charge on each is of the same kind and magnitude as the charge on the electron.

(3) In 1919, Rutherford showed that when atoms of nitrogen gas
are bombarded by alpha particles some few of the atoms send
out charged particles with a unit positive charge and a mass
about equal to that of a hydrogen atom.[2] Such particles have
been given the name proton and are interpreted as identical
with the nucleus of a hydrogen atom, i.e. a hydrogen atom with-
out its single electron. The name *proton* (which might be inter-
preted as "first thing") indicates the feeling that here we are
dealing with a primary constituent of matter. It is plain that
the proton in this experiment, having a positive charge, can
have come only from the nucleus. It must therefore form part
of the nucleus.

4. It was possible then to form a simple theory of the make-up of
the nucleus, and a simple wide-reaching theory is balm to the soul of a
harassed scientist. Long ago we had agreed to adopt for our unit of
atomic mass one-sixteenth of the mass of the oxygen atom. In terms
of this unit, the proton like the hydrogen atom, has a mass of quite
nearly one.

The nucleus therefore, so runs the theory, contains a number of pro-
tons equal to the mass number of the atom. And by the mass number
we mean the whole number which is nearest to the mass of that par-
ticular atom as it has been determined by the mass spectrograph.
These protons would give the atom the proper number of units of
mass but would as we shall see do more. Outside the nucleus are a
number of electrons equal to the atomic number of the element. This
atomic number ranges between 1 for hydrogen and 92 for uranium.
Four higher numbers, running up to 96, have been assigned to four
artificial elements the products of atomic research of the war years.

But these electrons of themselves would not be sufficient to balance
the positive charges in the nucleus. To take an example: The ordinary
atom of oxygen has a mass number 16 and atomic number 8. It would
therefore have 16 protons in its nucleus with a total positive charge of
16 units while the 8 electrons would bring in only 8 units of negative
electricity. It was necessary therefore to suppose that the nucleus con-
tains eight electrons which balanced the extra eight units of positive
charge and made the whole atom neutral. Another example on a larg-
er scale: The ordinary uranium atom has a mass number 238 and
atomic number 92. It must, according to this, have in the nucleus 238
protons and outside the nucleus 92 electrons. Therefore into the
nucleus must be put a number of electrons equal to the difference be-
tween 238 and 92, namely 146. The whole atom, though alive with
electric charges, is now neutral.

These electrons in the nucleus would act as a valuable cement, so

to speak, helping to hold together the protons against their strong mutual repulsions. They would serve to explain why beta rays, i.e. electrons come from the nuclei of certain atoms. This view of the structure of the nucleus recommended itself for its simplicity and the fewness of the necessary particles.

5. But in 1932 a disturbing intruder appeared on the scene. It had been found that certain very penetrating radiations were given off when alpha particles fell upon beryllium and certain other light atoms. On account of their power of penetration these had been assigned to the category of gamma rays; they were of the photon character, a sort of electromagnetic wave. But some awkward facts fitted badly into this theory.

Chadwick of Cambridge,[3] after an intensive study of these penetrating rays, concluded in 1932 that they are not radiations at all, but a new kind of particle, without any charge and having a mass about equal to that of a proton. This neutron was proved to have a high velocity and this, together with its lack of any charge, would explain its great penetrating power. For having no charge it could not be repelled by the positive charges on the nucleus. A moderate speed would enable it to enter the nucleus. It could be slowed down only by making a direct hit on one or more of the nuclei of the atoms it met. This neutron evidently comes from the nucleus for only there is so great a mass to be found. Further experiment showed that it could be obtained from any out of a wide range of atoms and that therefore it must form a part of these atoms. The presence of such a particle had been suspected before on theoretical grounds but these experiments brought it into the light. And it has indeed remained in the light; for it had a sinister role to play in the atomic bomb which has for the time overshadowed its beneficent action.

6. This new arrival must therefore be given a place in the nucleus. It was necessary to revise the former theory of its structure. This necessity gave the physicists no particular chagrin for they hold, with Joseph J. Thomson, that a scientific theory is a policy not a creed. It must as far as may be conform to facts and be able to interpret facts. The nucleus would then have a number of protons equal to the atomic number, i.e., equal to the number of extranuclear electrons and a number of neutrons equal to the difference between the mass number of the atom and the atomic number. Thus the nucleus would have the proper charge and the proper mass. We note however, that the electron is now excluded from a place in the nucleus; this in spite of the fact that beta particles are known to come out of the nuclei of some atoms. But it is not believed that they exist there as such. This new theory proved to be quite satisfactory and a good guide in future research.

7. The same year 1932 brought to light the positron[4] a particle
with a mass exactly equal to that of an electron and a charge of the
same magnitude but of opposite sign, that is, positive. This too must
come from the nucleus. It was soon proved that it has a very short
life time, appearing only under special conditions. Further research
showed that a positron and an electron could come together, fuse their
individualities, so to speak, and issue as a free radiation, a photon or
electromagnetic wave of high energy. Their particle character was
apparently lost or submerged. As a converse of this a high energy
photon could disappear as a photon and produce a pair of particles,
an electron and a positron. One aspect of this might be considered
disconcerting, since it indicated that one of our "solid" particles could
shed its grossly material nature and take on the character of a dis-
embodied burst of radiation energy.

8. So elusive is the positron that although many ingenious research
workers had since 1897 sought its acquaintance it retained its incog-
nito until 1932. Then it was found and recognized by the special tech-
nique of the cloud chamber as applied to the study of cosmic rays.
This special by-product of the study of cosmic rays earned for Carl
D. Anderson the Nobel prize in physics in 1936.

9. Here it is necessary to discuss that most valuable tool of research
the "cloud chamber."[5] This is a closed chamber containing dust-free
air or other gas such as argon and a quantity of water or alcohol. For
simplicity's sake let us take water. The space soon becomes saturated
with water vapor. Then the air space is expanded and thereby cooled.
The air has now a superabundance of water vapor for that tempera-
ture; but, unless the expansion has gone beyond a certain limit, this
does not condense into visible droplets. But if now electric ions are
produced throughout the gas the water vapor will condense round
these forming a visible cloud of exactly the same essential nature as
those concerning which Shelley rhapsodizes.

We are more interested in the case where the ions are produced
only in certain lines or spots. If a charged particle, let us say an alpha,
travels through the gas it will create a vast number of ions in its wake.
For it will by its attraction pull many electrons out of the atoms
through which it passes creating equal numbers of positive and nega-
tive ions. If moisture condenses about these as it will if the space is
supersaturated they show up as a white streak which may be photo-
graphed.

The possibilities here are many: (1) The path followed by the parti-
cle can be indicated. (2) If we apply a magnetic field whose lines of
force are perpendicular to this path the ion-producing particle can be
made to travel in a curve forming part of a circle, for it is charged.
The direction of this curvature tells us whether the charge on the par-

ticle is positive or negative. Also, since the radius of curvature can be measured we can get valuable information about the energy and momentum of the particle. (3) The ionizing power of charged particles differs widely. The alpha particle with two unit charges and a large mass and velocity produces a great number of ion pairs per centimeter of its path making a thick heavy line which can be quite readily distinguished from that produced by a proton. On the other hand the electron and the positron, being equal in mass and in magnitude of charge produce thin lines which are essentially alike except in the direction of curvature in a magnetic field.

10. The applications of the cloud chamber have been legion, but it has been of special use in the study of cosmic rays. The ingenuity of physicists has devised means for taking pictures automatically immediately after some particle whose trail is being followed has passed through the chamber. The pictures so taken can be subjected afterwards to detailed study. At the same time the apparatus can be arranged so that it will not respond to particles in which the investigator is not interested.

We may have heard a great deal about the Geiger counter which seekers for uranium or other radioactive material carry about with them. Essentially it consists of a closed cylindrical tube of thin-walled glass having within it two electrodes which are kept at a difference of potential of about 1000 volts. One of the electrodes is a copper cylinder fitting closely inside the glass and the other is a small central wire which is positive. When a cosmic ray or other charged particle enters the tube it ionizes the gas within producing electrons which move swiftly towards the central wire producing more ions on their own account. A tiny momentary current will flow which being amplified will cause a click in a loud speaker or will actuate some counting mechanism. When the lucky prospector finds his Geiger counter, placed near a mass of rock, giving fifty clicks per minute he knows that the rock will bear investigation.

Now these Geiger counters have been used with excellent effect in connection with the cloud chamber.[6] Two or three or more of these can be placed in line some above and some below the cloud chamber and so connected with the operating circuits of the chamber that only particles coming from a definite direction and passing through all of these Geiger tubes can cause expansion in the chamber and actuate the photographic camera. Such an exposure would be called a couter-actuated exposure produced by a coincidence circuit.

A great deal of curiosity has been excited by the extremely penetrating portion of the cosmic ray radiations. Some particles have such high energies that they are able to pass through all the miles of atmosphere and still penetrate to great depths beneath water or through

rock. So the cloud chamber has been pressed into service to take pictures on the highest mountains as well as at sea level and at stations in between. Balloons have been sent up to great heights with automatic recording devices.

In 1935 Anderson and Neddermeyer of the California Institute of Technology made 10,000 counter-actuated exposures at the top of Pike's Peak at an elevation of about 14,100 ft. About one out of every hundred of these pictures showed the path of a particle which could only with great difficulty be attributed to a proton and was certainly not an alpha particle or a positron. It seemed much more probable that it represented a particle whose mass was much less than that of a proton but at least a hundred times as great as that of an electron.

11. Announcement of this was made in 1936.[7] It was not long before various investigators reported similar findings and it was plain that a new sub-atomic particle had swum into the ken of physicists. Since its mass lay between that of a proton and that of an electron, the lightest particle known, it was called a mesotron (from the Greek word mesos, middle). It has now more generally come to be called a meson. These particles cannot be either electrons or positrons since they are too penetrating. They cannot be protons for the degree to which they are curved by a magnetic field is of quite the wrong value indicating a mass much less than that of the proton.

12. Hence we must class the meson as a new elementary particle. Some of them are positive and others negative. They are very evanescent particles, having an average half-life of about 2.3 millionths of a second. This will help to explain why they were not noticed earlier. When they break up, "decay," the electron is one of the products. We shall say more later about the mass of these particles.

13. It might well be asked how we can assert anything about the half-life of such a frightfully evanescent particle. The story is an interesting one. Presumably they are produced from atoms of matter in the higher levels of the atmosphere. We can observe the number of vertical counts in a coincidence circuit first when it is used at a high altitude and then (the same instrument) at a lower altitude. Quite naturally the number in the second case is distinctly less, since the air intervening between the two altitude levels must absorb some of the particles.

The absorbing power of air for various particles has long ago been measured. We can now repeat the experiment at the higher altitude putting in the path of the rays a layer of lead or other material which is equivalent in absorbing power to the mass of air intervening between the two levels. We should expect that the reading would now agree with the original reading at the lower altitude. As a matter of fact it was decidedly higher.[8] This shows that the falling off in the

number of mesons that reach the lower altitude is due not only to absorption by the matter between but also to a natural mortality of the mesons, their tendency to change over into something else, into other forms of energy. With reasonable estimates of the speed of these particles we can make a calculation of their half-life.

14. Very naturally the meson, this fleeting and mysterious particle of such extremely high penetration, has attracted the attention of many ingenious research workers. The findings of their intensive work are yet incomplete and will naturally be amplified and made more definite. The following points seem to be pretty well established:

1. There are at least two types of mesons:
   (a) Those which appear mostly at high altitudes. These react strongly with protons and neutrons, hence their half-life is very short. Their mass has been estimated at about 300 times the mass of an electron. (One observer gives about 282 and another about 315.) This π meson as it has been called, decays into a μ meson.
   (b) The μ mesons, which react slowly with protons or neutrons, are very penetrating. Their mass is about 200 times that of the electron. Their half-life is from two to three microseconds. They arise solely as a decay product of the π or heavy meson, while they themselves decay into an electron and another particle which has been called the neutrino.

2. There is some good evidence that mesons of still higher mass sometimes appear. Their traces have been found in the emulsions of photographic plates as will be explained below. These κ mesons, as they have been called, are estimated to have a mass about 1100 times that of an electron. The trace of one of these shows it as causing disintegration of a nucleus, sending out two swift mesons and a slow negative meson which was soon captured by a nucleus with the emission of two heavy particles.

   Some evidence has appeared for the existence of mesons of mass 700 to 800 times that of an electron. When the whole family history of the mesons is disclosed we shall probably find a great deal of diversity within a certain unity.

3. While mesons were first observed only in connection with cosmic rays they have lately been produced artificially. The new 184-inch cyclotron at Berkeley California can speed alpha particles to an energy of about 380 million electron volts; these were allowed to strike a carbon target. Negative mesons coming out of the target were observed in 1947. In 1948 two groups of positive mesons issuing from a carbon target subjected to the same radiation were observed and studied.[9] The mesons of these

two groups have masses of about 300 and 200 electron masses respectively and are presumably the same as the π and μ mesons described above.

15. What is the role of the meson in the atom? It turns out to be, apparently, an extremely important one.

We have seen that protons and neutrons are the building stones of the atomic nucleus. How can they be made to cohere, to form a stable structure? For the several protons, being all positively charged, repel each other fiercely and apparently the neutrons being uncharged cannot neutralize that repulsion.

16. It is difficult for us to realize the magnitude of that repulsion. A gram of uranium would be an extremely slight amount not one whose disappearance would worry the vigilant curators of our atomic energy resources. This 1 gram of uranium 238 would contain about 25×10^{20} atoms. Each of those atoms contains 92 protons. If we were to separate (in imagination, for it is only in imagination that it could be done) these protons into two compact groups which we would place at a distance apart equal to the upper estimate of the diameter of the nucleus, namely 10^{-12} cm., and if these two groups were allowed to act on each other by their un-neutralized repulsions they would repel each other with a force of about 3 times 10^{42} tons, a force which only the imagination can make an attempt to visualize. If we wished to be satisfied with 1 milligram of uranium the force of repulsion would still be 3×10^{36} tons.

Since we find that the nucleus of any atom in spite of its wildly repelling protons is a peaceful and permanent structure, enduring indefinitely, we can only conclude that there are within it extremely powerful attracting forces. Of the exact character of these we know very little. But it is necessary to suppose that they are forces which are extremely powerful at close range but fall off very rapidly with increasing distance. Thus two protons coming within a certain distance of each other would be very strongly attracted with a force which would completely neutralize and overcome their repulsion.

17. Now it is believed that the meson plays a large part in producing this attraction. Supposing that both the proton and the neutron have a mesonic charge g, we could speak about the mesonic field throughout which this charge could produce mesonic force and about the mesonic potentials existing there.[10] Just as the mutual electric potential between two bodies each with electric charge e would be $-e^2/r$ where r is the distance between them, so the mutual mesonic potential between two mesonic charges g is $-\epsilon^{-\eta r}g^2/r$. The presence of the factor $\epsilon^{-\eta r}$ shows that we are assigning a law of force for the mesonic charges which differs radically from Coulomb's law for the

action between two electric charges. The η is a new universal constant whose value lies between 0.3 and 0.5×10^{13} and which is the reciprocal of a length.

According to this the mesonic potential at a distance $r = 10^{-13}$ cm. between the two mesonic charges g would be about $\frac{2}{3}$ of g^2/r while at the distance $r = 10^{-12}$ (about the diameter of the nucleus) it would be only about $1/55$ of g^2/r. It therefore diminishes very rapidly with increasing distance, and with distances equal to the diameter of an atom it has to all intents vanished. This force would therefore be a typical short-range force very powerful at distances less than half the diameter of the nucleus, utterly insignificant at distances equal to the diameter of the atom. Such a force can at minute distances counteract the strong repulsion acting between the protons.

18. We might picture the proton and the neutron as exchanging a meson alternately between them. From one point of view the neutron and the proton could be considered as essentially the same kind of particle one being in a different energy state from the other. In recognition of this they are both called nucleons. Thus if a neutron received a positive meson whose charge is equal to that of an electron it would be equivalent to a proton but might be considered as a neutron in a higher level of energy. At any rate the continual exchange of a meson between them gives rise to "exchange forces." The meson field is the "exchange field" carrying electric charge to and fro between proton and neutron.

19. It is interesting to recall that in 1935 Yukawa,[11] a Japanese scientist, considering the short-range forces existing between neutron and proton and trying to account for them, postulated the existence of particles having a mass about one-tenth that of the proton. In that same year Anderson and Neddermeyer took on Pike's Peak the pictures which on being examined indicated the presence in cosmic rays of such a particle.

20. A very interesting method of detecting and investigating mesons (and other charged particles) has been worked out largely through the labors of Professor Powell, Dr. Occhialini and others at Bristol University, England.[12] It involves the use of photographic plates with an emulsion which has been treated in a special way. A charged particle passing through an emulsion will ionize many atoms along its path. When the plate is developed these will show as a line of black spots or grains. A dense line with many grains to the centimeter will be proof of strong ionization. An electron will leave a thin trail. The character and the length of the trail will give a good index of the particle that produced it.

The preliminary work in this type of investigation is easy indeed. It consists in placing the photographic plates wrapped up in black

paper on a shelf in some place of high altitude leaving them there for about three weeks, allowing the cosmic rays and their products to work their will upon them. The results will be more abundant if the plates are placed so that the emulsion is in a vertical position.

The heavy work will come after the plates have been developed. They must be observed over their whole surface and through the whole depth of the emulsion for the tracks of charged particles. This must be done by a rather high-power microscope. When one of these tracks is found, its course must be mapped, its density must be measured. If the track is found to branch each one of the branches must be followed and mapped. Naturally the particles whose course ended in the emulsion will be of greatest interest. The degree of ionization as revealed by the density of the track must be recorded for each branch. All this adds up to hours and days of tedious work even after the worker has familiarized himself with the technique. The great advantage of the method is that one plate may contain as many records as would be given by a long succession of individual pictures taken by the cloud chamber.

Naturally particles of different kinds leave their tracks on these plates. Among them are mesons, of at least two different kinds. Indeed the heavy meson was first clearly recognized in some of these tracks; and in general the findings agree quite well with the results obtained by other methods. There are tracks in which the π (or heavy) meson, after a short journey, gave rise to a μ meson. This, being less affected by particles near which it passed, has travelled a much greater distance before coming to the end of its range. There are evidences that the μ meson at the end has produced an electron which leaves its own distinctive trail.

The Bristol group of workers report having found a total of 644 meson tracks which end in the emulsion of the plates. About two-thirds of these plates had been exposed in the Pyrenees at an altitude of 9240 feet while the rest came from an altitude of about 18,100 feet in the Bolivian Andes.

The photographic emulsion method has been applied to the examination of tracks made by the mesons which have been produced by the use of the cyclotron. The pile of plates is placed in the path of the beam of mesons issuing from the target which is being bombarded by the high speed alpha particles. A powerful magnetic field can be applied in a direction perpendicular to the path of the mesons. In this fashion the positive mesons can be separated from the negative and each group made to fall upon its own pile of plates. The cyclotron is a far more copious producer of mesons than the cosmic rays.

It is quite certain that the meson will loom large in future discussions of nuclear structure. A number of groups of able research work-

ers are studying it experimentally and we can anticipate that they will shed some much-needed light on its nature and its action in the nucleus.

By way of a summary of what we have been saying we may append the following table of the fundamental particles.

TABLE OF FUNDAMENTAL PARTICLES

Particle	Kind of Charge	Number of Elementary Units of Charge	Mass (in Atomic Mass Units)
Electron	Negative	1	.000548
Proton	Positive	1	1.00758
Positron	Positive	1	.00548
Neutron	None	—	1.0090
Alpha Particle	Positive	2	4.0097
Beta Particle	Negative	1	.000548
Meson	Pos., Neg.	1	~0.11
Neutrino	No direct evidence		?

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HIGH SCHOOL EXHIBITS AT THE CAS&MT

A feature of the Central Association meeting to be held at the Edgewater Beach Hotel in Chicago, Illinois, on November 25 and 26, 1949, will be student exhibits. Any member having outstanding exhibits, made by their pupils, in the fields of Mathematics, Biology, Chemistry, Physics, or Elementary Science is invited to bring these exhibits to the meeting.

Advance notice of the type of exhibit with information as to the space required must be sent to Dwight L. Barr, 5525 Glenwood Ave., Chicago 40, Ill.

Are you taking time to read at least one educational journal regularly? Try SCHOOL SCIENCE AND MATHEMATICS this school year.

HELPING THE ELEMENTARY SCIENCE TEACHER

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During two years of supervision of elementary science in the Cleveland Schools, several devices have been tried to help the teachers in this field. A well planned science curriculum was inherited. It, therefore, seemed advisable to devote the major part of one's time to helping teachers make the program function more effectively and give greater satisfaction to both children and teachers.

To get an over-all picture of the situation, the upper elementary science teachers in the 114 elementary schools were visited. As a result of these visits, the outstanding problem seemed to be a feeling of insecurity in the subject matter on the part of many teachers. Some teachers are afraid to teach science. Others say they are not interested in science. This is probably due to a lack of knowledge of the subject matter.

The lack of subject matter background is not the fault of the elementary teacher. Her preparation has included largely cultural courses, psychology, and techniques of general elementary teaching. When she is confronted today with a typical science course of study, she finds that she needs zoology, botany, astronomy, geology, chemistry, and physics. Of course, the concepts in these fields are simple at the elementary level, but the vast area covered is none the less frightening. It looks impossible to her.

To get a more accurate knowledge of the background of these teachers, a questionnaire was sent to the upper elementary science teachers asking them to list all the science courses they had taken in either undergraduate or graduate work. The following is the result:

Master's Degree in Science						3
Bachelor's Degree in Science						16
Number taking 8 science courses						10
"	"	7	"	"		6
"	"	6	"	"		8
"	"	5	"	"		9
"	"	4	"	"		21
"	"	3	"	"		15
"	"	2	"	"		13
"	"	1	"	"		12
"	"	0	"	"		32

TOTAL TEACHERS REPORTING......... 145

Note: The majority of teachers have either a Bachelor's or Master's Degree, but not in science.

The survey also showed that 47 teachers had taken their science courses prior to 1935.

In order to help the teachers, it seemed necessary to overcome a fear of science teaching and to help supply some of the necessary subject matter.

To accomplish the above objectives, the first step was to try to create the feeling that the supervisor should be considered a consultant and co-worker. Obviously, in a system as large as Cleveland it is impossible to get around often enough to give specific help to all individuals or to get acquainted with all teachers personally.

To bring the Headquarters office closer to the teacher and to help her feel that the department is interested in her personally, a mimeographed bulletin is issued two or three times a semester. This bulletin contains several short reviews of good teaching techniques observed in various classrooms. It is the hope that these reports will encourage the teacher who has done a good job and will pass on ideas to less experienced teachers.

Another feature is a digest of any science meetings attended by the supervisor or the high points from some current professional journal. New science books for both teachers and children are reviewed.

Many cities have clubs and groups interested in such science activities as lectures and field trips. The bulletin carries the announcement of all such meetings. Taking advantage of the opportunities offered by a Museum of Natural History or an Audubon Club is one way a teacher may add to her knowledge of subject matter.

Teacher contributions to the bulletin are encouraged.

During the school year 1947–1948, two meetings with competent speakers were arranged. To stimulate an interest and furnish some subject matter background for the astronomy unit, a meeting was held at the Observatory under the direction of the astronomer. Since this proved effective, a similar one was planned with the head of the Weather Bureau as speaker. This meeting set the stage for the fifth grade weather unit.

For the year 1948–1949, another plan was tried. The Geology unit, involving a simple knowledge of rocks and minerals, fossils, and the forces changing the earth's surface, was found to be one of the most difficult units for the teachers. To help overcome this difficulty, two optional meetings were planned for teachers new to the teaching of science. The first dealt with the classification of rocks and minerals, and the second with the Geology of the Cleveland region. The response was gratifying and the teachers themselves asked for the next three meetings. Electricity, sound, and aircraft were the units presented.

The meetings aimed to present simple, usable material and point out the essential concepts to be taught.

The supervisor conducted all meetings except the one on aircraft. A teacher with unusual ability in that area conducted the meeting. It is hoped to continue the meetings this year and to capitalize further on the special abilities of the teachers. The meetings will continue to be optional on the teacher's part.

Teachers were asked to learn with the children in their classes and not to be afraid to say, "I don't know, but we shall try to find out together." Children respect a teacher for being honest rather than bluffing.

Of course, some of the teachers new to the teaching of science need help on methods to lead them into an activity type of teaching where the children handle materials, make observations, and perform experiments. To help these teachers, the supervisor taught a series of four or five lessons so that the teacher could observe different techniques and also see the growth from day to day. A series of lessons is of more help to the teacher than an isolated one.

A number of years ago, Park Protection Clubs were organized in many elementary schools. These science clubs have conservation and protection of the natural features in the various school neighborhoods as their major emphasis. Monthly meetings of representatives from each building are held at the Cleveland Museum of Natural History. Good speakers are brought to the children at these meetings. Groups of children often contribute to the program. The information carried back to the classroom by the representatives stimulates the conservation work in the various classrooms.

The Board of Education stations a teacher at the Natural History Museum. Her function is to take classes in the Museum for lessons which correlate with the classroom work. These experiences enrich not only the children's background but also furnish subject matter for the teacher. The Cleveland Zoo is another contact which helps both children and teachers.

The primary teacher also needs guidance. The subject matter is simple, but she needs help in presentation. To help the large numbers of primary teachers, meetings were held after school in fourteen districts so that no teacher had to travel a great distance.

At the beginning of the meeting there was a brief discussion of the general philosophy of science teaching. The remainder of the hour was devoted to a demonstration of the use of many types of visual aids helpful in science teaching at the primary level. The teachers themselves showed ways in which they had used visual material. They were assisted by one of the staff from the Division of Visual Education.

It is hoped to continue and expand these plans so that the teachers

will gradually lose their fear and insecurity and begin to have enthusiasm for the subject as well as for its teaching.

A FORMULA FOR DETERMINING HYBRID RATIOS

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Examining the ratios produced in the first four sequences of hybrid crosses we find the following ratios:

Monohybrid cross produces a ratio of:	3:1
Dihybrid ratio produced is:	9:3:3:1
Trihybrid ratio produced is:	27:9:9:9:3:3:3:1
Quadrihybrid ratio produced is:	81:27:27:27:27:9:9:9:9:9:9:3:3:3:3:1

It will be seen, therefore, that the first number of the ratio is determined from the number of characteristics being crossed in the hybrid used as a power on the base three:

$$
\begin{aligned}
\text{Monohybrid} \quad (1) &= 3^1 = 3 \\
\text{Dihybrid} \quad (2) &= 3^2 = 9 \\
\text{Trihybrid} \quad (3) &= 3^3 = 27 \\
\text{Quadrihybrid} \quad (4) &= 3^4 = 81
\end{aligned}
$$

The second number in the ratio is again determined as a power on the base three, but the degree of the power is one less than the first term:

$$
\begin{aligned}
\text{Monohybrid} \quad (1) &- 3^0 = 1 \\
\text{Dihybrid} \quad (2) &- 3^1 = 3 \\
\text{Trihybrid} \quad (3) &- 3^2 = 9 \\
\text{Quadrihybrid} \quad (4) &- 3^3 = 27
\end{aligned}
$$

The third number appearing in the ratio, similarly, is determined on the base three with a power of two less than the number of characteristics being crossed, and so on until the last term of the ratio (3^0) which is one is reached.

The above discussion enables us to find the numbers appearing in each ratio, but does not tell us how many of each number occurs. To obtain this frequency of occurrence let us examine the ratios of the first four hybrid crosses again.

1.	3:1
2.	9:3:3:1
3.	27:9:9:9:3:3:3:1
4.	81:27:27:27:27:9:9:9:9:9:9:3:3:3:3:1

If the number of times each number in the ratio is grouped we have an interesting relationship:

1.
$$3 : 1$$
$$(1) - (1)$$

2.
$$9 : 3:3 : 1$$
$$(1) - (2) \quad (1)$$

3.
$$27 : 9:9:9 : 3:3:3 : 1$$
$$(1) - \quad (3) - \quad (3) - \quad (1)$$

4.
$$81 : 27:27:27:27 : 9:9:9:9:9:9 : 3:3:3:3 : 1$$
$$(1) - \quad (4) - \quad (6) - \quad (4) - \quad (1)$$

The frequency of these numbers alone set up shows us that we have an excellent example of Pascal's Triangle:

$$1$$
$$1 - 1$$
$$1 - 2 - 1$$
$$1 - 3 - 3 - 1$$
$$1 - 4 - 6 - 4 - 1$$

Knowing the power of each number in the ratio (on a base of three), and the frequency of occurrence of each number makes it possible to develop a general formula:

$$3^n : 3^{n-1} : 3^{n-2} : \cdots : 3^0$$
$$(a) \quad (b) \quad (c) \qquad (1)$$

In which n is the number of pure strain, dominant characteristics being crossed with a similar number pure strain, recessive characteristics; and $a, b, c, \cdots 1$ (the last term) is the frequency of the numerical occurrence which is figured from Pascal's Triangle. This is always the $n+1$ row of the Triangle since the figure is the first row is one.

In application of this formula let us now determine the ratio for a pentahybrid:

$$3^5 : 3^4 : 3^3 : 3^2 : 3^1 : 3^0$$
$$(1) \quad (5) \quad (10) \quad (10) \quad (5) \quad (1)$$
$$243:81:81:81:81:81:27:27:27:27:27:27:27:27:27:27:$$
$$9:9:9:9:9:9:9:9:9:9:9:3:3:3:3:3:1$$

This formula, of course, applies only in cases where there is complete dominance of all characteristics crossed. The first number in the ratio contains all the dominant characteristics, the second group of numbers contain all the dominant characteristics minus one, the third group contains all the dominant characteristics minus two, until the last number of the ratio is reached which contains all the recessive characteristics.

SPECIAL TRAINING FOR TEACHERS OF ARITHMETIC*

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Both pre-service and in-service special training can be of value to the teacher of arithmetic. The teacher training institution merely starts the training. If the training program is successful at all it must take into full consideration the training which comes with teaching experience and planned in-service activities. In this discussion of special training for arithmetic teachers I hope there will be found suggestions helpful to all arithmetic teachers. While the most direct interest in the discussion may be for those responsible for college teacher training programs, new associations and new emphases, if not new ideas should be suggestive to all teachers interested in arithmetic.

Furthermore, any consideration of special training for elementary teachers must be based on recognition of the tremendous task confronting the elementary school teacher. Desirable special training in so many areas makes nearly impossible demands on her time. Too, because she is a generalist, general education so important for all teachers is even more important to her. I hope careful analysis of the proposals which follow will show that the many opportunities and responsibilities of the elementary teacher are fully recognized.

TRAINING GOALS

As a starting point for examination of arithmetic teacher training programs a statement is given of certain teacher training goals, including some which are quite broad and to which time will permit only brief reference. These are goals for the experienced teacher or graduate student in elementary education. They are desirable goals for any teacher whether or not he earns credits leading to a degree. Formal school courses should make achievement of these goals easier. It is however not my intention here or at any other point to draw a sharp line between training in colleges and training on the job.

While the core professional courses offered for all teachers, and teacher institutes and meetings certainly give much attention to goals 1 and 2, these goals should also be re-emphasized and interpreted in the professional courses in special areas and in-service training programs in special areas. In this paper goal 3 will be taken up last since it serves as a bridge for our change of focus from a graduate program to an undergraduate program.

The teacher training program should help arithmetic teachers:

1. Develop for themselves a philosophy of education which recognizes the importance and responsibility of teaching as a profession; the role of the teacher as school and community leader; the general education responsibility of the school and that the role of arithmetic is determined largely in terms of this general education responsibility; the ideal of frequent search for new ways to serve the school and society.

2. Identify teaching problems and determine relative importance of problems, and to learn methods of seeking solutions of educational problems, particularly as related to arithmetic.

3. Better understand mathematical concepts which have most direct bearing on the arithmetic of the schools, and appreciate the necessity for the teacher of arithmetic to broaden his intellectual interests to include many areas of knowledge and understanding.

4. Become familiar with sources and means of using current professional literature and the older writings of direct interest to arithmetic teachers, including books, periodicals, and reports of significance, both in pedagogy and subject matter; and to know the historical background in literature and experience for current educational bias and soundness.

5. Acquire some knowledge and understanding of the total mathematics curriculum for grades 1–10 and the possibilities of each year's experiences to contribute to a continuous growth in mathematical power.

6. Learn to use a variety of means of enrichment of learning experiences including applications in daily living and to other areas of instruction, visual aids, good procedures of evaluation, and reference to cultural and social significance of mathematics.

Every school administrator and teacher should be familiar with the recommendations on teaching mathematics and science to be found in *Manpower for Research*, a report of the President's Scientific Research Board (7). The Report lists reasons for current weaknesses in accomplishment in arithmetic and recommends changes in teacher training and teaching procedures which should aid in overcoming those weaknesses. The Report also emphasizes the desirability of a sound philosophy of education. I count the recognition of the great responsibility and great opportunity of the teacher an essential part of such a sound philosophy. Teachers of today through their influence on boys and girls can make a new world tomorrow. What greater role has any man?

That the place of arithmetic is determined largely in terms of the

general education responsibility of the school is in no way better emphasized than the wise selection of the title for the Sixteenth Yearbool of the National Council, *Arithmetic in General Education* (11). Teachers need to be reminded that these general objectives include both social needs and maximum development of the potentialities of the individual. An adequate program in arithmetic cannot be based on social needs alone unless the use of that term includes as social needs development of attitudes, ideals, approaches to problem solving, sometimes known as mathematical modes of thinking.

Teaching Problems

Teachers in my class last summer had little difficulty in identifying problems. The list and frequency of occurrence is of interest. One-third of the teachers of arithmetic when asked to list problems which they would like to study, included individual differences. The next topic in order of frequency was procedures in teaching problems and problem solving. Teaching fractions was in third place. Two teachers listed the retarded child and two listed enrichment for superior students.

Teaching techniques for grades 7 and 8 was listed as a critical problem by two teachers. Among problems listed by a single teacher were planned versus incidental arithmetic, problems of supervision, grade levels versus areas of instruction, place of drill, remedial teaching, use of texts and workbooks, teaching of division, teaching of borrowing, curriculum for grade 7, prognostic value for algebra of arithmetic tests, and methods of teaching arithmetic which cause difficulty in algebra.

An attempt was made to help each student see the relative importance of his problem in relation to his whole teaching situation. Each individual was allowed special credit for successful procedures he was able to develop in seeking possible solutions of his problems.

It was observed through group investigation of the implications of the meaning theory of instruction that acceptance and use of the meaning theory on all grade levels provided a new hope of solving many problems in teaching arithmetic. Since this study was seen to relate to so many of the specific problems listed it became the central core of the course. A similar situation might emerge in individual teacher study or in-service groups.

The problem of individual differences has important implications for the training program. An individual contract plan in arithmetic makes it much more difficult for the teacher to teach meaning rather than mechanics. Students must be led to discovery of new relationships and generalizations through directed experiences. It is almost impossible for the teacher to provide these experiences for one or two

individuals at a time. I should not want to see a class in which at no time was the interest of all focused on the same topic. After exploratory early experiences further development will be necessary in groups or on an individual basis, but even for this the groups need to be as variable as possible. Most any Johnny sometimes gets interested in an idea which may have caught his fancy and he thereby belongs with the better students for that particular idea.

Individual differences must be cared for in terms of the progress and depth that individuals may achieve with a particular topic. Short division can be taught when students are ready to discover it which for some of course is never.

One aid in providing for individual differences would be identification of minimum understandings and skills with which different individuals and groups may be permitted to leave a topic and go to the next. State and city curriculum committees could make their classroom materials of greater value if they indicated possible minima on which both group and individual instruction could be based for any grade or level.

Professional Literature

All teachers need to become familiar with professional literature in their own area of interest. For the upper grades SCHOOL SCIENCE AND MATHEMATICS and *The Mathematics Teacher* can be recommended. *The Elementary School Journal* should of course be available in the school library and if every teacher of arithmetic could read or have access to a half dozen of the articles in the valuable summary of writings on arithmetic that appears in each November issue, better practices in arithmetic would result.

The 100 selected references to be found in the *Sixteenth Yearbook* (11) would provide an excellent bibliography from which any teacher could choose good professional reading. The *Tenth* and *Sixteenth Yearbooks* of the National Council should have been studied by all teachers of arithmetic. Furthermore, these references require discussion if the teacher is to get the most from them. Although the date of the *Tenth Yearbook* (9) is 1935 it still could provide a source of valuable discussion material for any teacher group.

In addition to the professional literature there is a large body of semi-popular writing on elementary concepts which could be recommended for elementary teachers. References (3), (5), (6), (8), (12), (14), (16) are illustrative of books which could well be in the arithmetic teacher's library.

Continuous Growth in Mathematical Power

Arithmetic power and competence are achieved by continuous

growth and development of the individual from kindergarten counting games to the algebra and geometry of the high school. The teacher needs to know the means of guiding development of both basic skills and concepts through the experiences of each year.

Place value is a concept which gives meaning to experiences at many levels. From the earliest numeration beyond the number 10 to polynomials in algebra, there is frequent reference to this device man has found so useful in reaching his present state of engineering ingenuity and business efficiency.

Place value is a unifying idea in counting, in the rearrangements which we call addition and subtraction, in the repeated additions and subtractions which we call multiplication and division. In roots and powers and short forms of computation and in checking, reference to place value will show relationships with earlier and more familiar concepts.

Many courses of study do a fair job of identifying the skills to be learned at each level. The contributions of the previous grade and the following ones to skills are not too difficult to recognize. Much less has been done in listing meanings which should be taught at the various levels and which should be developed through the experiences of more than one year's work. A list of meanings to be taught, which will be valuable both for use in teaching and in teacher training, is to be found in *Arithmetic 1948* (1). W. B. Storm has listed 54 understandings that can be learned in arithmetic through division. The list has been prepared in co-operation with the Committee of Seven, and was discussed at the Arithmetic Conference at the University of Chicago last summer. It is not intended to be a complete list, and some might not express the meanings in the same form. Nevertheless such a list of meanings takes its rightful place with the lists of skills previously considered the core of instruction in arithmetic. We have all been concerned with the 81 addition facts and the 810 higher-decade combinations of which 176 are used in carrying in multiplication. When our arithmetic course of study is described in terms of skills, the teaching is apt to be mechanical and the product merely a temporary tool. Teachers would be much more likely to teach meanings if they are given assistance in identifying the meanings. Teachers in groups or as committees in schools or systems, or even in courses, would find it a profitable experience to try to list meanings they will teach.

One of Storm's understandings which is important to all levels is "understanding the significance of checking." Attention must be given to checking and its significance in the training program. More important still for those learning arithmetic is the recognition of the reasonableness of a result. This practice should be cultivated in all

situations no matter in what way the result may be obtained and in what social situation it may arise.

One of our gravest faults in the mathematics curriculum has been the large gap which has existed between the program of the elementary and secondary schools. That is being corrected by an extension of the general mathematics of grades 7 and 8 to grade 9. All of us have been too satisfied to teach concepts and skills outlined for our particular grade to the complete neglect of the kind of experiences that have come before or will come after. In mathematics we have actually done a better job of anticipating the needs of the next course than of relating experiences to the immediately preceding one. If high school teachers knew the problems and goals of arithmetic teaching they could do a much better job and do it more efficiently. The *Manpower for Research* (7) report emphasizes the importance of the 12-year perspective.

In many city and state curriculum committees elementary and secondary teachers are getting together to work on common problems. This co-operative work and the widespread acceptance of the meaning theory are the most promising trends of today in mathematics. Schools and teacher training institutions must plan programs which guarantee that teachers of arithmetic become familiar with the goals in arithmetic of the grades which precede theirs and at least two which follow. If the institution which started the training of your teachers failed in this respect then it is all the more important that your school make provision to remedy the deficiency.

Important Teaching Aids and Procedures

At the recent Wisconsin Education Association meeting two speakers made a real contribution for teachers in their presentations of two important uses of arithmetic. These were arithmetic needs in retailing and in industry. Both presentations were made with references to the special training programs necessarily maintained for prospective workers in these areas. The speaker for retailing called attention to the fact that everyday computations with fractions and decimals are not as simple as we sometimes are led to believe when she made reference to the cost of $4\frac{3}{4}$ yards at $1.95 per yard; or $10.95 plus luxury tax of 20%, plus sales tax of 2%. Direct references to community uses of arithmetic should be made in all teacher training programs.

Teachers in training should have opportunity to see and discuss all recent arithmetic films and film strips and older productions of particular merit. Experienced teachers' questions last summer about a new arithmetic film revealed the desirability, as nothing else had, of study of the meaning theory even for those who have taught. Films provoke good discussion in teacher training groups, even those films

which are merely suggestive of procedures that the teacher could better use herself than show by film.

Films are only one of many visual aids useful in arithmetic. Teacher or even student prepared visual materials are always of great value. The preparation of such aids by teacher training groups not only serves to provide the teacher with materials which may be of later use but even more this work requires original planning and concrete experience in preparation for direction of meaningful teaching.

Evaluation in its broadest sense has come to be recognized as an important component of all good teaching. Professional courses for all teachers will certainly include instruction in this area. Measurement of understanding rather than mechanics is a real problem in arithmetic instruction and attention must be given to it in the teacher training programs at all levels.

In the Wisconsin curriculum conferences this fall there was considerable interest in units for use in arithmetic which cross over subject matter lines. The current acceptance of science as a part of the elementary school curriculum opens a new area for correlation with arithmetic. Much study needs to be given to possibilities of correlation of arithmetic and science particularly in the upper grades. This would be a very appropriate topic for special investigation by the Central Association. At some future date the Elementary Mathematics section could profitably devote an entire program to this problem.

SPECIAL TRAINING IN MATHEMATICS

Point 3 of the goals for teachers includes the desirability for the teacher of arithmetic to broaden his intellectual interests to include many areas of knowledge. This, as has been observed, is not special training for arithmetic teaching and yet it may be more critical in this area than any other of the elementary school curriculum. The breadth of intellectual pursuits which have a direct interest for the teacher of arithmetic can scarcely be exaggerated. In undergraduate training, the new programs of integrated liberal studies, given such impetus by the Harvard Report (4), make it more possible for the elementary teacher to have broad cultural experiences and to develop interests which should enrich many later teaching situations. In one respect these programs are an attempt on the college level to get away from sharp subject matter divisions. They are an important step forward as far as general education for elementary teachers is concerned.

Such programs cannot carry mathematics instruction in a satisfactory manner, except as a separate course, any more than incidental learning can provide an adequate arithmetic program. Colleges and universities are also considering modifications of the traditional first

year mathematics courses to make more certain their contribution to general education. Courses like the Type I and Type II general mathematics for the Junior College, described in the Fifteenth Yearbook of the National Council, *The Place of Mathematics in Secondary Education* (10), could make real contributions to the professional as well as the general education of arithmetic teachers. In one the emphasis is on broad concepts and certain general mathematical procedures and their applications. In the other the core is social uses, particularly statistics and part of what is commonly taught as mathematics of investment. A course of the latter type has been offered at Southern Illinois University for a number of years. Its original intent was for arithmetic teachers. It actually turned out to be a popular course for many other groups as well.

In a four-year course for elementary teachers I would strongly recommend the inclusion, at least on an elective basis, of a year's sequence of special training for teachers of arithmetic, if possible for four hours a semester. At least one semester of this sequence should be taught by members of the mathematics department and be described as a mathematics course. Teachers of the course should be selected on the basis of their special interest in and sense of the value of teacher education on both the elementary and secndary levels. If at least one semester of this course can count toward general education requirements, so much the better. A course in mathematics planned for general education, like those just described, might be made the first semester of this sequence. It could be much more desirable, however, if a course could be planned and offered especially for elementary teachers.

A selection of topics for such a course planned for elementary teachers might be made from the following list: the number system, including historical background; use of symbols in problem solving; tables and graphs; elementary topics in statistics; ideas of approximate computation; logarithms; recreational arithmetic; compound interest; annuities and elementary topics in life insurance; trigonometric ratios in indirect measurement. This list is not unlike that recommended in General Mathematics, Type I, in the Fifteenth Yearbook (10). A systematic development of algebra would not be necessary but algebraic principles would naturally be used in a study of the topics listed. Frequent references to historical background and significance should be included.

When teachers study techniques for teaching meaning in arithmetic they discover that too limited understanding of basic mathematical ideas is a major handicap in an attempt to understand relationships and logical developments in elementary number situations. Undergraduate students in training are usually much more deficient in this

respect than experienced teachers, and some course in mathematics of the kind just described is of even greater importance in the undergraduate training program. It does not seem impossible to me that in large systems individual deficiencies in basic mathematics might be made up with the assistance of a competent teacher of mathematics in the system. Such a program should of course be one based on ideas new to those taking the course and one of complete emphasis on meanings—not remedial work on skills, old or new.

Remedial work with meaning, if necessary to bring all prospective teachers up to a defined minimum level of achievement should be provided before any special training in mathematics. The check list of minimum essentials of the Commission on Post-War Plans (13) might be used as a minimum level of achievement. This list also can serve later as a teacher's guide to some of the principal goals in the total arithmetic program and hence should be known to all teachers of arithmetic.

Even though the prospective teacher may have made a good record in a first semester's college mathematics course of the type discussed above, he may still have little real understanding of some of the basic concepts of elementary arithmetic which he will be called upon as a teacher to teach meaningfully. Direct attention must be given to these concepts in the teacher training program. Books like the recently popular *Elementary Arithmetic* by Buckingham (2) and the older *Arithmetic for Teacher Training Classes* by E. H. Taylor (15), one of the pioneers in the move to teach arithmetic to prospective teachers of arithmetic, can be very useful as guides or texts in this study. It is my recommendation that this material be studied in close association with the work on methods of the second semester of the sequence for arithmetic teachers and that it also be brought into the first semester's work as it is related to the topics under consideration.

The second semester's course should be devoted to problems in teaching arithmetic with very special emphasis on teaching meaning. This course can be taught so much more efficiently and effectively if students have had some recent experiences in elementary but basic mathematical concepts. In this course there probably should be two basic texts, one of which is a methods text and the other a text on the meanings of arithmetic. Viewpoints and teaching aids discussed in the preceding sections are also important for the undergraduate student and should be brought into the course as time and previous experience of the students permit.

Through the year's sequence the prospective arithmetic teacher must obtain clear understanding of the basic mathematical concepts so that he will be able to lead children to think meaningfully and independently in situations involving number. Too often children

merely cultivate the habit of dependence upon rules set down by others for them. Children see number and form and some order in the world around them. So far as possible their classroom experiences should be similar to their out-of-school contacts with number. But this is not always easy in the process of leading children to see relationships and seek generalizations. The teacher must be able to introduce situations which will create for the student the need to see meaning and relations.

All teachers in training need to re-emphasize in their own thinking two aspects of teacher responsibility. One is to provide for the present and anticipated (as nearly as is possible) future needs of their students. The second responsibility is to provide the student an introduction to goals, ideas, ideals, relationships, and possibilities that he would not easily find in his everyday experiences present or future.

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The YEAR BOOK of the Central Association of Science and Mathematics Teachers will be out soon. Is your name on the list of members?

TEACHING FORMULAS AND EQUATIONS BY THE USE OF LABORATORY EXPERIMENTS

FRED T. WEISBRUCH

William Cullen McBride High School, St. Louis, Mo.

It would hardly seem necessary to point out the importance of formulas and equations in the course in high school chemistry. Much has been said and written about the over-specialization of the high school chemistry course and unfortunately, there does exist today, a tendency to make the course one of the "appreciation course" variety. But the fact still remains, that the fundamentals of any subject must be mastered before a true appreciation of the concepts of that subject can be obtained. This is especially true of chemistry where the mastery of symbols, formulas, valence and equations constitute the core of the subject matter. Obviously these are the fundamentals, without which it is practically impossible, either to speak the language of chemistry or to understand simple chemical reactions. One might as well attempt to teach a modern language without constant and insistent drill work on vocabulary and verb forms. What vocabulary is to a modern language, formulas and equations are to chemistry. Master them and the language of chemistry becomes comprehensible, and the science of chemistry takes on a new and intelligible meaning. Ignore them and for all the student understands, one may just as well be speaking Greek.

And yet there are a suprisingly large number of teachers who discount the importance of such work in the high school chemistry course. They seem to prefer leaving such specialized teaching to the college, thus depriving the student of one more opportunity of exercising his intelligence. Is it fair to take away from the student every challenge to his mind because a sincere effort is required to master this "dry and un-interesting" memorization of formulas and valence? I firmly believe that even a mediocre student is sufficiently endowed mentally to master these fundamentals if they are presented to him as a challenge. Often, much of what the student learns of formulas and equations, is a reflection of the mental attitude of the teacher toward the importance of these fundamentals. In the chemistry department of McBride High School we have always considered the mastery of symbols, valence, formulas and equations as one of the minimum essentials of the course. We do not consider a student capable of carrying out laboratory work intelligently until he has mastered these fundamentals. Various devices such as flash-cards, valence and formula card games are resorted to in order to arouse interest in the subject and lend a little zest to the process of memorizing. When all

other devices fail the del'nquent student or the lazy student is submitted to a process of testing and re-writing his corrections, until by a "supreme mental effort" he finally succeeds in overcoming his mental inertia. Since this occurs early in the year it soon convinces the student that chemistry can be interesting because it is a challenge to his mind and above all that chemistry does require persistent work and the development of certain very definite study habits. Unfortunately it is for some the first time in their high school career that they have come up against the necessity of mental discipline. This in itself, I believe, would justify the insistence which we give to the writing of formulas and equations.

Once the teacher accepts the fact that one of his main objectives of the course is the mastery of formulas and equations, the question arises as to the proper method of presenting this topic to his students. Certainly it should not be softened up because it is difficult. Equally as certain is the fact that more attention and preparation is necessary on the part of the teacher in order to present formulas and equations in an attractive and readily understood manner. Therefore wherever the laboratory can serve this purpose it should be used to the utmost advantage.

Certain limited phases of this topic can be presented to the student in the laboratory in such a manner that the visual demonstration of chemical reactions may be a help to the rather dry memorization of long lists of valences, symbols and formulas. However, let no misapprehension arise at this point—the student must memorize the usual valences and symbols.[1] But the mastery of them can be an achievement and a mental challenge, and the use of the laboratory can help in a great measure toward this end.

I have devised several experiments which we use at McBride to facilitate the teaching of formulas and equations, through the visual observance of certain reactions. The success of these experiments, judged by the increased interest and level of attainment of the students has encouraged us to include them as regular laboratory work, along with the standard experiments. We feel that the extra week spent on valence and formulas and the two extra laboratory periods devoted to formulas, equations and the law of conservation of matter, has a marked influence on the students' attitude toward the chemistry course and an increase in understanding of chemical reactions.

The following is a sample experiment that is given here in the hope that other teachers may find the use of it profitable in arousing interest and making chemical reactions more intelligible to the student.

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[1] For a tested listing of required formulas and equations for high school chemistry, see Brauer, O. L., "What To Expect of the High School Student in Chemical Formula and Equation Writing," *J. Chem. Educ.*, 5, 304 (1928).

It should be noted that, since we teach the laboratory work by the "semimicro method," quantities called for in the experiment are usually in drop quantities or not in excess of one or two milliliters.

CHEMICAL FORMULAS AND EQUATIONS

Problem. How Are Chemical Equations Used To Express a Chemical Reaction? To What Law Must a Chemical Equation Conform?

Introduction.

You have learned that elements may be represented by symbols and that a chemical formula shows the *exact* chemical composition of a chemical compound. Since chemical reactions take place between the atoms of elements and compounds, these same formulas may be used to show how a chemical reaction takes place. The substances which react (reactants) and the new products formed as a result of the chemical action are expressed as a *chemical equation.* A chemical equation shows the nature of the reactants, the products formed and the number of atoms of each element taking part in the reaction. The total number of atoms in the products formed is always equal to the total number of atoms in the compounds reacting. Likewise, by the law of conservation of matter, the total weights of the reactants is always equal to the total weights of all the products formed as a result of the chemical action. If the total number of atoms *is not* the same on both sides of the equation, then the equation must be *balanced* until they are the same.

(1) Write the name of the compound, the chemical composition of which is represented by the formula, Na_2CO_3 _____

(2) Give the number and the names of the different *kinds of atoms* in this compound. _____

(3) How many atoms are represented by the formula for aluminum sulfate, $Al_2(SO_4)_3$? _____ Zinc Nitrate?

---

(4) Balance the following equation so that there are the same number of atoms on each side of the arrow. $H_2 + O_2 \rightarrow H_2O$

If the number of atoms on both sides of the equation is not the same, place numbers in front of the formula for any compound or element until the number of atoms is the same. *Under no condition* is a subscript number to be changed.

Experiment.

PART I. Using Chemical Equations to Represent Chemical Reactions.
1. *Reaction of Silver Nitrate with Hydrochloric Acid.*

Place about 1 ml of silver nitrate in a small test tube and add to it 10 drops of hydrochloric acid. Note whether or not a chemical change has taken place. Examine the precipitate.

(5) Does the precipitate resemble either of the reactants?_____

(6) Name the white precipitate. _____ Formula? _____

It is evident that a chemical reaction has taken place and that this reaction can be represented by a chemical equation showing the reactants, the new products and the total number of atoms involved in the chemical change. The reaction is represented by the following equation.

$$AgNO_3 + HCl \rightarrow AgCl + HNO_3$$
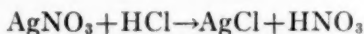
(7) What is the total number of atoms on each side of the equation? _____

(8) Since the number is the same, the chemical equation is said to be _____ and therefore, represents a true chemical equation.

2. *The Equation for the Reaction of Cupric Bromide with Ammonium Hydroxide.*

Place a spatulaful of cupric bromide in a small test tube and add 1 ml. of water. Dissolve all the salt by shaking the tube. When the salt has dissolved, add ammonium hydroxide, a drop at a time until the precipitate forms. This precipitate is cupric hydroxide.

(9) Describe the precipitate. _____ _____

(10) Does it resemble either of the original reactants? _____

Some of the formulas in the following equation for the above reaction are incorrect. When properly written, the equation represents the chemical reaction between cupric bromide and ammonium hydroxide.

$$CuBr + NH_4OH \rightarrow Cu(OH) + NH_4Br$$

Examine the equation and, by checking the valence of each element, correct all formulas that are written incorrectly. Remember that a chemical equation cannot be balanced unless the *exact* composition of the compound, as shown by the formula, is given.

(11) When the formulas are correctly written, count the number of atoms on each side of the equation. Are they the same? _____

If the number of atoms is not the same on both sides of the equation, change the number of molecules of any compound necessary to balance the equation. *Do not change subscript numbers.*

(12) Now write the balanced equation for the reaction of cupric bromide with ammonium hydroxide._____

3. *The Equation for the Decomposition of Mercuric Oxide.*

Heat a spatulaful of mercuric oxide in a test tube until part of the compound is decomposed and you are able to identify *all* of the products of the reaction.

(13) Name the two products that are formed when mercuric oxide is decomposed. _____

(14) What is the formula for mercuric oxide?_____

(15) Write the *balanced* equation for the above reaction. _____

4. *The Equation for the Reaction of Lead Nitrate and Potassium Chromate.*

Place 10 drops of lead nitrate solution in a test tube and add to it, 10 drops of potassium chromate.

(16) Name the precipitate in the above reaction and describe its appearance. _____

(17) Write the correct formula for each of the reactants. _____

(18) Write the balanced equation for the above reaction, after making sure that all formulas are correctly written. _____

PART II. A Comparison of the Molecular Weights of the Reactants and the Products Formed in a Chemical Reaction.

Remember that the number of atoms on the two sides of a chemical equation must be the same. Since the number of atoms are the same, it follows that the total weights of all the atoms on one side of the equation must be equal to the total weights of all the atoms on the other side of the equation.

(19) Write the equation for the reaction between potassium iodide and lead nitrate._____

Check the solubilities of the two products of the reaction in the *Table of Solubility* in your laboratory manual.

(20) Which product is insoluble? _____

Dissolve a spatulaful of potassium iodide in 1 ml of water. Be sure that all the salt has dissolved and then add 10 drops of lead nitrate.

(21) Name the precipitate and describe it._____

(22) Compare the precipitate with the original reactants used to bring about the precipitate._____

It is evident that the new insoluble product is _____

and that the product which remains in solution is _____

Using the *Table of Atomic Weights*, look up the atomic weights of each one of the elements which you have written on the left side of the equation.

(23) What is the total of all these atomic weights? _____

(24) Is this number the same as the total of all the atomic weights represented by the right-hand side of your equation? _____

(25) According to the law of _____, the total weights on the two sides of a *true* equation must be _____

Part III. The Chemical Equation and the Law of Conservation of Matter.
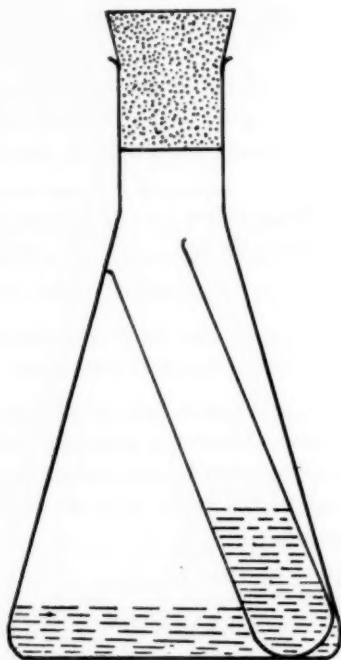

Fig. 1.—Law of Conservation of Matter.

Obtain a 125 ml. flask with a cork to fit. Place 1–2 ml. of lead nitrate solution in the flask. Place 1–2 ml. of potassium chromate solution in a small test tube and carefully lower the test tube into the flask so that it rests on the bottom of the flask. (See Fig. 1.) Cork the flask and place it on the pan of your balance. Adjust the weights until

the flask is perfectly balanced. Invert the flask so as to mix the contents, and note the yellow precipitate of lead chromate. Compare the precipitate with the original reactants. Are they the same? Replace the flask on the balance.

(26) Has there been a change in weight? _____

(27) What law does this demonstrate? _____

(28) Write the balanced equation for the reaction.
_____

(29) Add the molecular weights of the compounds on each side of the equation. What is the total weight for each side of the equation? _____. A _____ chemical equation exactly duplicates the nature of the _____, the products formed and the actual _____ of all compounds taking part in the reaction.
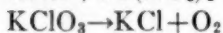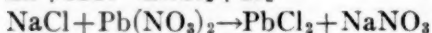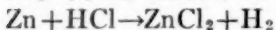
HOW WELL DO YOU UNDERSTAND THIS EXPERIMENT?

1. What is the valence of X in the compound, X_3O_5?
2. What is the valence of Fe in the compound, $K_3FE(CN)_6$?
3. Write the correct formula for each of the following compounds.

Potassium nitrate	Sodium chloride
Ammonium carbonate	Calcium hydroxide
Silver nitrate	Barium sulfate

4. Balance the following equations.

$$K + Cl_2 \rightarrow KCl$$
$$Fe_3O_4 + H_2 \rightarrow H_2O + Fe$$
$$Zn + HCl \rightarrow ZnCl_2 + H_2$$
$$NaCl + Pb(NO_3)_2 \rightarrow PbCl_2 + NaNO_3$$
$$KClO_3 \rightarrow KCl + O_2$$
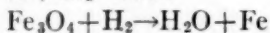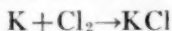
5. Write balanced equation for each of the following reactions.

Decomposition of mercuric oxide.
Calcium plus water.
Magnesium plus sulfuric acid.

A BULLETIN FOR MATHEMATICS TEACHERS

A second printing of the pamphlet, *A Mathematics Student—To Be or Not To Be?*, is now available at a charge of ten cents per copy. Separate copies of the accompanying chart, suitable for bulletin board use, are available for five cents each or ten for twenty-five cents. Orders should be sent to Professor Phillip S. Jones, Department of Mathematics, University of Michigan, Ann Arbor, Michigan.

THE SUN-DIAL

PRINCIPLE UNDERLYING CONSTRUCTION

Morley F. Fox

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In Figure 1.

Let P be the earth's north pole. It is the sub-polar point of the celestial sphere.

Let R be the radius of the earth, with numerical value of 1 in following equations.

With P as center and radius R, draw circumference to represent the equator. Divide it into 24 sectors of 15° each and to the points of section draw lines from P to represent meridians. Considering the sun to be at equinox, it passes from point to point at intervals of one hour.
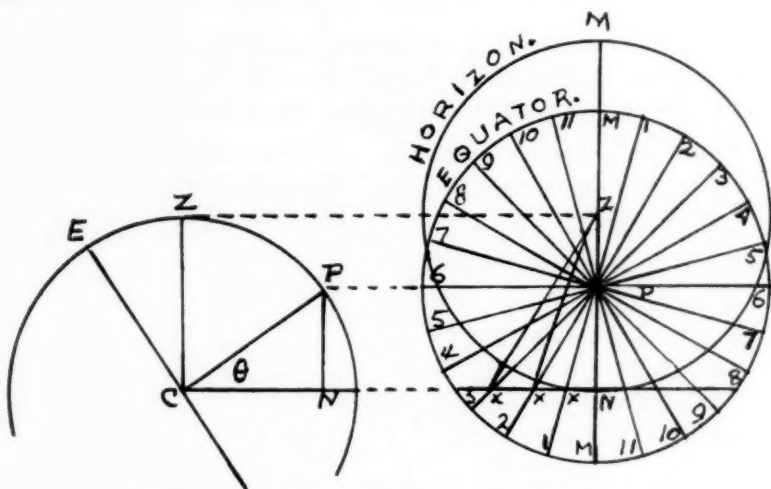
FIG. 2   FIG. 1

Select a meridian, say PM, as principal meridian passing through a given place on the earth of which the latitude is designated by θ and let the point Z in that meridian be the sub-zenith point of the place.

With Z as center and R as radius, draw circumference to represent the horizon. It bisects the equator and intersects the several hour-lines. The intersection of the horizon with the principal meridian at N is the north point of the horizon. NP is the elevation of the celestial pole, subtending the angle θ of the latitude. At N draw tangent to the horizon, intersecting the hour-lines at xxx etc; and from Z draw the several lines Zx (of which but two are shown).

Reflecting that the gnomon of a horizontal dial has its base in the

meridian at Z and its style pointing to the north celestial pole, the subtended angle being that of the latitude, consider that:—

$$\text{Tangent} \angle NZx = \frac{Nx}{NZ} = \frac{Nx}{R} = Nx. \tag{1}$$

$$\text{Tangent} \angle NPx = \frac{Nx}{NP} = \frac{\text{tangent} \angle NZx}{NP} \quad \begin{array}{l} \text{by substitution of value} \\ \text{of } Nx \text{ in (1)} \end{array}$$

whence the tangent of $\angle NZx = (NP)$ (tangent $\angle NPx$). $\tag{2}$

In Figure 2.

C is the center of the earth; Z the sub-zenith point; CN is in the axis of the horizon, being in the plane of the tangent to the horizon; P is the earth's pole, sub-polar point of the celestial sphere; NP is elevation of the pole above the horizon and angle θ is angle of the latitude. R is the radius of the earth, as in Figure 1.

$$\text{Sine } \theta = \frac{NP}{CP} = \frac{NP}{R} = NP.$$

Putting this value of NP in equation (2), the tangent of $\angle NZx$ = sine θ tangent $\angle NPx$. That is, the tangent of the angle between the gnomon and the edge of its shadow equals the product of the sine of the latitude and the tangent of the sun's hour-angle. This is true of a dial that is horizontal and oriented exactly north and south.

A DEVICE FOR EFFECTIVE PRESENTATION
OF DIAL METER READING

JOEL J. RHEINS
Corona Junior High School, Corona, New York

One of the areas of application of Mathematics to daily life is the reading of dial meters present in the home. These include gas, electric, and water meters, that operate on the decimal principle.

It is desirable in teaching this topic for each pupil to have at his seat a dial of his own. However this procedure is beyond the resources of most schools. The following activity has been found to produce an adequate personal teaching aid for each child. It is, moreover, in its own right, a worthwhile geometric practice exercise. It was used successfully in a very dull (Median I.Q. 68) mixed seventh year class.

Divide an ordinary small sheet of oaktag into quarters; each sheet will then be approximately 4″×8″. Rule a faint line parallel to the long edge and two inches from it. Locate the following points on this line: the midpoint and points two inches on each side of the mid-

point. At each of these points describe a circle with a radius of one inch. Inside the circumference of each circle mark off the integers from 0–9 spaced equally with the "0" at the top and the "5" at the base side. The "0" and the "5" points can be located by erecting perpendiculars to the midline at the centers of the circles. The location of the other points may be fixed by visual approximation. If
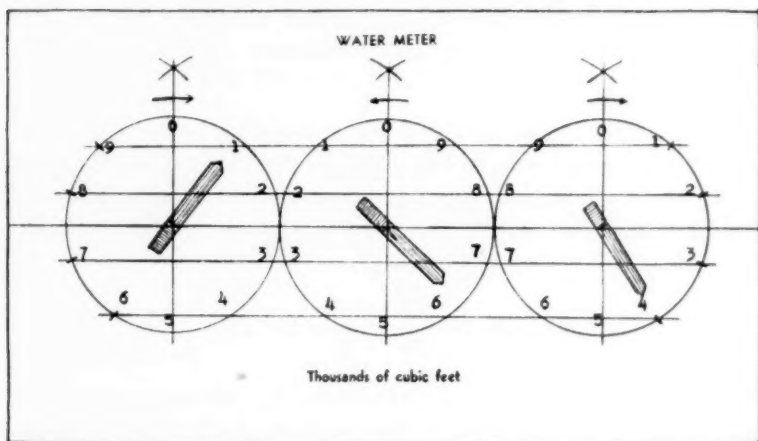
the "1-2-3-4" points are located on the integer circle and the "6-7-8-9" points on the hundreds circle the rest of the points may be determined by the intersection of the circles with lines joining the following point pairs: "1–9," "2–8," "3–7," and "4–6."

The pointers are simply made. Through each center push a ¾ inch brass round head paper fastener. Open out the fastener leaves; one will be pointed the other will be rounded. Fold back the rounded end upon itself, halving its length. The dial is now ready for use. It may be labeled as the appropriate type of meter being studied at the time.

One of the boys in the class reproduced the dial on a wood panel. He substituted a strip of metal nailed to the center of each circle for the brass paper fastener as the dial pointer.

RIDER TV MANUAL VOLUME 2

Continuing its pace setter policy for the radio servicing industry, John F. Rider Publisher, Inc., announces TV Manual Volume 2 is currently available at Rider distributors.

Exceeding the original estimate of manufacturers scheduled for inclusion, TV Volume 2 covers the products of 67 manufacturers, including TV boosters.

The combination of Giant, Triple, and Double Spread pages supplemented by the TV—HOW It Works, book Volume 2, makes for time-saving, profitable TV servicing. The total number of pages in TV Manual Volume 2 approximates 2,300.

Priced to sell at $18.00, its ownership is a sign of successful servicing.

A DEFENSE OF THE PROJECT METHOD

HERMAN O. HOVDE
Loveland, Colorado

Writing in the June 1949 issue of SCHOOL SCIENCE AND MATHE-MATICS Fred T. Weisbruch, in his article "Glorifying the High School Project" made an attack on this method of teaching in which several statements were made which call for an answer. Admittedly the project method, like any other human device has faults, and these were pointed out in the article.

There is a basic concept, which called for the satire Mr. Weisbruch used, that will not stand up under present school conditions. This idea is that our high schools are for the group of our population which learns the abstractions of the old type school program, for example Latin, Geometry, and other subjects taught from the standpoint of learning a mass of facts. This is an untenable idea, because we now have the entire population of the country of the age levels from 14 to 18 approximately, in our schools.

Since the entire population is in school, it is no longer possible to conclude that activities associated with the high school of the nineteenth century will solve our present difficulties. In addition there has been much study made of the processes of learning, and the old type of school program did not take into account this material, naturally enough, since it was unknown when the program was begun.

"Many teachers," says Mr. Weisbruch on page 441 of the issue quoted, "will defend the project idea because it 'Stimulates and creates an interest' in the subject. If this is true then a number of teachers have been sorely neglecting a very important phase of the education of their students. I doubt very much if there is any stimulation in projects of the above type. Rather it is confusing knowledge with activity."

As one who has used the method under criticism, even in the way that is so ridiculed—that is projects of cutting pictures from magazines—I feel that the neglect of education for many students is a fact and that the opponents of the project method need to look to their philosophy and practice of teaching. Even if the project method consisted of the satirized "cutting and pasting" there is ample evidence in the schoolroom of any average American city to show observing adults that limited understandings are possible with such elementary methods and that for many of our pupils, compelled by law to attend school, this may be a satisfying activity and may result in definite progress.

Many students need the stimulation that comes from the accomplishment of a simple task successfully to develop interest in the sub-

ject in its deeper ramifications, and for some it may be the understanding accomplished by this type of activity is as far as progress can be expected. It appears that to develop a simple understanding of scientific concepts may be all that is in the range of some students, and such a limited understanding is better than no knowledge. In the nineteenth century it was adequate to say to a pupil "Stop school, and go to work." Now we cannot say that. Instead the community says "Here are our children. Teach them."

The simple activities of a "project" also lead into more complex and worthwhile activities which develop the individual greatly. In use of the project for a period of years it has become apparent that such activity will lead many pupils into other activities which reach a higher level of understanding and appreciation. Many students who have done work of this kind are now citizens of the community taking part in adult activities with no more training in science than that offered in the high school physics and chemistry classes. Many of them entered the science courses with fear and doubt as to their success, having heard of the difficulties to be expected. If the course had been taught from the standpoint of strict knowledge and rigid "mental discipline" one of three things would have occurred: 1, those people would have dropped the course in a short time, 2, enrollment would have declined as that group turned to other courses, or 3, failure, with its bad effects on pupil and community. The project method eliminated these choices for the less able students, referring to ability as scholastic success, and allowed, at the same time, for a program of activities in the class for those who were able to do, and could profit by, intensive work. Thus all levels of ability were served.

The major difficulty with the project method brings to mind Shakespeare's comment on another matter: "The fault, Dear Brutus, is in ourselves, not in our stars, that we are underlings." In the project method of teaching the fault most often lies with the teacher, not with the method or the pupil. It is more difficult to teach by projects than to assign a set amount of text matter, recite, conduct a routine experiment in the laboratory and so on. Proper teaching by the project method will require of the chemistry teacher familiarity with all phases of the subject, commercially and industrially, as well as theoretically. Such familiarity is not held by the author, but for the student in class it may be at times, better that there are some areas in which the instructor is not well informed, because this uncovers his human stature and demonstrates to the student that teachers have something to learn.

Teaching by the project method, if properly done, will develop the originality of the pupil. Laboratory experiments, mentioned by Mr. Weisbruch as being required to provide a learning ground, are and

have been standardized until much work is routine. Allowing the project method activities in this part of the course will eliminate such unstimulating work, because the projects suggested by the students will require establishment of procedures, research into methods of carrying on suggested experiments, and solution of other problems which will take co-operation and thought on the part of both instructor and student. This supplies the kind of laboratory activity called for by Mr. Weisbruch.

A consistent error, on the part of the teacher who condemns the project method, on the basis that it interferes with learning of "knowledge" is that knowledge is the goal of teaching. Perhaps that is a worthwhile goal for a very few schools, but not many of our children are to be prepared for life in which this is necessary. The need, for most of our students, or rather pupils, is the attainment of skills, attitudes and appreciations which will be the living values of their school life.

Another error in the thinking is that a choice must be made between knowledge and the project method. This is far from true. Experiments carried on at Columbia University indicate that in chemistry laboratory those who did the work, i.e., those who had an opportunity to develop skills, were not handicapped in knowledge, but rather seemed to have more of it.

As teachers, and citizens of the country, too, we must face the fact that all the people of this country of high school age are now attending school. Physically this is established, but mentally there has been no adjustment whatever for many, many teachers. The "standards" by which the selected group, attending high school during the last century, were judged can in no way apply to the mass of our present high school population. These criteria are deader than the carrier pigeon. Then high school education was for those who were to enter professions, such as law, the ministry, or other college requiring occupations. It was judged adequate to give training for college, and the demands of professors were listened to and heeded. High school graduates were accepted, if qualified, and the college claimed to have educated them. Now it is necessary for high schools to educate the entire people, and it is vital to say to the colleges "We have given the essentials of living in a democracy to people. Now you educate those who are to specialize, and we hope you will give as much attention to the teaching as you do to the materials to be learned, as we in high school have had to do."

In another statement in his article Mr. Weisbruch says of the science courses offered in high school: "Possibly we have watered down the science course until the student no longer has any desire for something which to him has lost its usefulness." From personal

experience in teaching physics and chemistry to today's high school students quite the opposite is true. Observation of students for fifteen years indicates many pupils who entered the science course with fear, having heard of its difficulty, When allowed to do work on projects, many of which were of the picture cutting level, they developed increasing confidence in their abilities, showed increased interest in the subject and finished creditably.

When the science courses require mathematics as a background and are taught mathematically, or rigorously, enrollment declines. Those schools who offer science courses with project type of teaching have had no difficulty in increasing enrollments. Many of the pupils who profit by the activities are now well thought of citizens in their community. Would it have been better to drive them from the physics and chemistry course entirely and have them live in our present world completely ignorant of the field?

The difficulty, where enrollments are low apparently is in the method of teaching, and not in the subject. Emphasis upon knowledge as a goal has failed to interest the pupils of today, but it is possible to interest them in the courses in science and to develop skills, attitudes and appreciations that are of value to them and to the school. As teachers we have an obligation to the country in providing for every one the greatest possible functioning of the abilities and skills of science. It is not our job to prepare engineers, and skilled professional men. We should concern ourselves in preparing all the youth to live with and operate the machinery and understand the functions of science in our day. This will give basic training to those who will go to college to prepare for engineering and other scientific fields.

In measuring achievement in science by enrollment there are hazards. Popularity in numbers is not a goal, but since all the people of high school age are now enrolled in school increasing enrollment in science means increased participation, and our goal should be science training for every one. To better care for the prospective technicians projects of more advanced skill, and understanding can be given.

Another defect with the idea of knowledge as a goal in teaching is that so much is forgotten. It would be extremely difficult to defend our school program, or the system of education itself from this standpoint. The mark of educated people is not an encyclopedic mind, but the tolerance, work habits, skills, attitudes and other habits, such as ability to attack and solve a problem, and to evaluate the reuslts. These then must be the goals of our teaching.

Personal experience will illustrate the value of activity. How often is it that an item to be remembered is called up by the incidents that are connected with it when it was first present in an activity, clues such as an unfortunate choice of materials leading to bad results, ac-

cidents which occurred and left indelible impressions, or the odor, feeling, sound, sight or other characteristics with which the desired item is connected bring it sharply into view. This contrasts with the statement "confusing knowledge with activity, learning with doing."

Doing is an integral part of learning. It is very possible to sit in a classroom and to learn the action of an automobile engine, the steps needed in starting the engine and in shifting gears, as well as the rules of safe driving, but no one at present would take a ride with a driver having only such training. Laboratory work with miniature equipment is not enough. To learn to drive you must do that activity, and under conditions that are met in ordinary life. This is the fundamental principle of the project method. Because it has been misused is not adequate reason for its condemnation.

In our education of today we need to deliver products, upon graduation that are integrated persons, because they have followed a program of activities which develop skills, attitudes, appreciations, and methods of attack on problems along with knowledge. No worse indictment comes to the school than the common statement of high school graduates that after twelve years of schooling there is no salable skill with which he can earn a living and take his place in society.

The time is now for a real change in our school program. The entire person must be educated in our schools. Let us meet the challenge and revise our thinking so that we perform the function that has been given to us. We who teach as well as those we have in our classes are living in a world of air transportation, radar and television, rockets, and nuclear power. These give us leisure and need for better appreciation of the energies in our control, as well as a demand for great skill on the part of every one in operating the devices which we commonly use. Let us teach all to live a full life, to be a useful part of a home and community and leave the highly technical fields to colleges. The trend is toward larger enrollments in college, which will assure an ample supply of professional people.

Shaffer in *Psychology of Adjustment*[1] states "The responsibility of the school is not limited to intellectual training. The child comes to school as a whole and it is impossible to separate his intellectual functions from his motives, emotions and social adjustments. This is especially true in modern education. A century ago, when terms were short, there was some justification for regarding the pupil merely as a passive learner. He obtained the majority of his most valuable experiences outside of school. Today the school demands a third, or more, of the pupils waking hours, compulsory education laws make every pupil attend, and an increasing number of duties formerly assigned to

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[1] Shaffer, L. F., *The Psychology of Adjustment*, Houghton Mifflin Co., N. Y., 1936.

the home are being assumed by professional educators. This greatly increased opportunity must be met with increased responsibility for the child's general welfare."

The above statement expresses the attitude that must be held by teachers if the needs of the school population are to be met. The way may not be clear in every detail, but there is adequate evidence to indicate that the method of teaching knowledge for its own sake, with no regard for the other values is no longer tenable, and that activity, which interests youth of itself holds the hope of developing an informed citizenry in this country.

Mr. Weisbruch in discussing the need for effort says, "An understanding that the mastery of any field is difficult and requires hard work, as well as an appreciation of the fact that hard work is rewarding of itself." With this there can be little quarrel, since it affirms the necessity of activity, which is the fundamental principle of the project method. If there is intended an implication that projects do not provide for hard work, or that since many projects interest students to complete absorption in their activity and therefore are not hard work, then there is lack of appreciation of project efforts.

Many students begin a project and develop, during their work on it, such absorbing interest that meals are forgotten, and every waking hour is spent in planning and carrying out plans. From such activities come hobbies of life long standing and in many cases even a life occupation results. These facts refute the implication that "knowledge by mental discipline" is the way to real education.

The project method will not solve all the ills of our schools. It has weaknesses, the worst probably being the great amount of material required at the command of the teacher, but it provides for individual differences, in ability and interests, it develops skills, understandings and attitudes as well as giving knowledge in accord with psychological principles and can therefore be used by any teacher sincerely interested in giving the best education to pupils in our schools.

ZIRCONIUM IS ANTIDOTE FOR PLUTONIUM POISONING

Encouraging experimental results have been obtained with the somewhat unfamiliar element zirconium as an antidote for the highly poisonous man-made atom-bomb element, plutonium. Zirconium injections into the bodies of plutonium-poisoned rats result in the release of the plutonium from the tissues where it was found, especially in the bones, and permit it to be eliminated by way of the kidneys. Considerable amounts of plutonium were thus eliminated even in cases where it had been tied up in animals' bodies for many months.

THE PLACE OF MATHEMATICS IN GENERAL EDUCATION

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It would be a repetition to review again in this paper the reasons for the changes in the American system of public education.[1] Fawcett[2] has stated them concisely and clearly in a recent publication. Let it suffice to say that criticisms have been made and let it be admitted that there is justification for many of the adverse comments. In particular it must be said that much mathematics teaching and many topics of courses in mathematics deserve the derogatory remarks which have been leveled at them. As a teacher of boys and girls I wish to take the position of working with administrators and other teachers (in different fields of specialization)—colleagues—in an attempt to plan a better program of instruction. As a specialist in mathematics I am interested in seeing and planning the ways in which this area of experience can contribute to the program. It is this position of cooperative attack on the problem rather than that of defense of the status quo in one's individual field which will lead in the direction of a reconciliation of the many points raised by parents, students, philosophers, psychologists, curriculum specialists, and teachers. Please recognize that these are not distinct groups of people; there is much overlapping, but each group has definite characteristics and opinions.

Rare, indeed, is the community which consults with parents and pupils on the question of curriculum revision. The usual procedure is to have a study made by a group of experts. If this is not feasible, a group of subexperts from the local educational scene study the problem. At no point during the research is more than casual consultation carried on with parents. On the other hand pupils are treated merely as the raw material for the new and better educational mill that is in the process of construction. This is not intended to depreciate the efforts that have been made to revise the curriculum; but in many cases the mistake has been a failure to consider the *total* picture of the educational scene. Occupying a prominent place in the foreground of this picture are pupils and parents; they should be given the consideration due them. Sometimes it seems as though the desires of the public, to whom the schools belong, are overlooked. When changes are undertaken, it is an excellent idea to find what parents

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[1] See *Mathematics in General Education*, A Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum, D. Appleton-Century, New York, 1940, Chapter I.

[2] Fawcett, Harold P., "Mathematics and the Core Curriculum," *The Mathematics Teacher*, XLII: 6, January 1949.

have to say concerning the plans; it will be found that their judgment is based on experience in living and upon a mature consideration of their own lives. Equally so, pupils who have their ways to make in life will want to have a hand in the plans that are being made for them.

On the collegiate level there has been much discussion of general education in recent years while on the secondary school level the core curriculum has occupied the center of the stage. What is *general education?* The Report of the Harvard Committee states general education " ... does not mean some airy education in knowledge in general ... nor does it mean education for all in the sense of universal education." The committee goes on to say exactly what it does mean by general education, "It is ... that part of a student's whole education which looks first of all to his life as a responsible human being and citizen."[3]

Although mention of general education and the core curriculum are carefully avoided in the report, "Education for *All* American Youth," yet its statement[4] is important to this discussion; the members of the commission write, "When we write confidently and inclusively about education for *all* American youth, we mean just that. We mean that all youth, with their human similarities and their equally human differences, shall have educational services and opportunities suited to their personal needs and sufficient for the successful operation of a free and democratic society." Notice how closely the statements from these two reports parallel one another. Two groups of people interested in the welfare of the student in his educational environment come from their separate studies to essentially the same conclusion concerning general education. Differences appear in the types of experiences provided for a student in order that he might attain these broadly stated objectives. However, the underlying philosophy remains the same; the *means* differ but the ends arrived at are identical.

Specific remarks with respect to the role of mathematics in a program of general education are the concern of this paper. What have these two reports to say on this matter? The members of the committee who wrote "General Education in a Free Society"[5] were thinking of the secondary school in its relation to college preparation. However, much of what they wrote applies equally well to the noncollege preparatory student because they never lost sight of their definition

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[3] Report of the Harvard Committee, *General Education in a Free Society*, Harvard University Press, Cambridge, Mass., 1945, p. 51.

[4] Educational Policies Commission, National Education Association of the United States and the American Association of School Administrators, *Education for ALL American Youth*, Washington, D. C., 1944, p. 17.

[5] *Loc. cit.* The following quotations come from the section whose title is *Mathematics in the Schools*, pp. 162-167.

of general education. To quote, "By the end of the seventh or the middle of the eighth grade every pupil should have acquired a reasonable facility in the language of arithmetic, the beginning of an appreciation of the number system, some competence in the solution of arithmetical problems, and some appreciation of the power of mathematics in formulating and solving problems in the real world. . . . every pupil should have learned the commoner facts of geometry. . . . The next stage in mathematical instruction, and the last for those students who are least apt in the subject, should convey an appreciation of the use of formulas, graphs, and simple equations, and should develop some skill in solving right triangles trigonometrically." The committee states that a pupil would have accomplished this learning by the middle of the ninth year; it recognizes that pupils who stayed on in high school to graduate might forget a great deal. To meet this objection the committee writes, " . . . it might be valuable to give them in the senior year . . . an introductory survey of elementary trigonometry, statistics, precision of measurement, and the use of graphs." To summarize this committee's point of view on the extent to which mathematics should be included in a program of general education, there is the sentence, "When a student has reached his limit of tolerance in handling abstractions, his *general* education in mathematics must also come to an end." Their thesis is that it is the *meanings*, the abstractions or generalizations, of mathematics which make its instruction worthwhile—that the manipulative techniques, the drilled upon processes, are soon forgot.

Carnahan[6] took as his point of departure in his paper a quotation dealing with the contributions that science could make to the building of an individual's philosophy of action. He asserts that the statement could have been made with equal justice about mathematics. He concludes that mathematics can make an outstanding contribution toward the achievement of the objectives of general education delineated in the report.

"Education for *All* American Youth" is devoted to analysis of the secondary school program for grades ten, eleven, and twelve. However, there is this comment concerning the earlier years, "Throughout the junior high-school period, . . . , the educational needs of pupils are sufficiently alike to justify a common curriculum for all pupils with ample provision for differential treatment of pupils within classes to take account of diversities of interests, aptitudes, and abilities."[7] This report considers general education as continuing throughout the secondary school and grades thirteen and fourteen. Provision

[6] Carnahan, Walter H., "Adjusting the Teaching of Mathematics to the Requirements of General Education," *The Mathematics Teacher*, XXIX: 211-216, May 1946.

[7] *Loc. cit.*, p. 230.

is made for it in a course the commission labels "Common Learnings" and a well worked out system of guidance. There is much merit in the schedule[8] that is presented. Study of it indicates there is flexibility enough to allow for individual differences in students and for particular vocational choices. A mathematics teacher, for example, need not fear that his subject would receive too little emphasis.

It seems a complete circle has been made and the starting point is again reached: What *is* general education? When does it stop, and special education begin? Of course, one might say that general education never stops; it goes on throughout life; as long as a man breathes he is engaging in some form of general education. Although this is a beautiful idea it is of very little help in efforts to solve the problem of the place of mathematics in general education. Will mathematics have a part in general education only through the ninth grade and thereafter be relegated to special or vocational education? This seems to be the implication of the Harvard Committee Report. Or will mathematics be a tool to be kept sharp and useful not only in one's educational experience but in life? There is evidence that leads me to believe this is the point of view of the commission which wrote "Education for *All* American Youth."

The courses called *General Mathematics* will not answer these questions. Such courses were designed to take care of the problem of the slow learner in mathematics, those pupils who could not (or would not) master the abstractions of algebra. There was no unifying philosophy of education to support them; no plan had been worked out which assigned these courses to their proper role in the educational scheme. As a result general mathematics courses were hopelessly stigmatized from the beginning. "In our effort to care for the great masses we . . . diluted our courses and talked down to the students in such a way as to tend to stifle intellectual development in those we need so desperately to furnish the leadership in science, industry, government, and other phases of human activity."[9]

Yet teachers of mathematics know that, "It is the responsibility of mathematics to develop ability to recognize and use quantitative data in the study of social problems."[10] Efforts of a piecemeal sort are made to put the general mathematics subjects on a firm foundation, to give them a character and a standing of their own. Success or failure depends largely upon individual teachers[11] or a group of teachers who approach the problem with both careful planning and enthusi-

[8] *Loc. cit.*, p. 244.

[9] Cameron, Edward A., "The Place of Mathematics in General Education," *The Mathematics Teacher*, XLI: 275, October 1948.

[10] Burr, Harriette, "Mathematics in General Education," *The Mathematics Teacher*, XL: 58, February 1947.

[11] See, for example, the paper by McCreery, Gene S., "Mathematics for All the Students in High School," *The Mathematics Teacher*, XLI: 302–308, November 1948.

asm. From what a member of such a group writes of the students, "They wanted this course . . . , because for the first time in their school experience they could understand their work in mathematics,"[12] one can understand and appreciate how well these teachers worked with pupils and parents to achieve such a reaction.

The answer to the problem of the place of mathematics in general education will not be found in easier mathematics or a dilution of the present courses in algebra and geometry. Because, as the Harvard Committee writes, "The pressure to make mathematics easier for students . . . is inclined to take the form . . . of developing it as a ritual of memorized formulas and procedures."[13] The report goes on to say, "It is unfortunately true that those aspects of algebra and geometry that are of greatest interest in general education are also more difficult to teach, and are much harder for the student to grasp, ..."[13]

Rather than this "talking down" to students and taking care of their problems by easier materials, a whole reorganization of the program in mathematics is needed. Such a reorganization is of no avail unless it is considered in the light of a school's philosophy of education. Only with such a philosophy as a frame of reference can a program be worked out which will satisfy the needs of pupils in general education. It has already been noted that there is a tendency among mathematics teachers to think of general education as all right for those who cannot do the regular work but algebra and geometry are the courses for those who can do it. I should like to assert that this is philosophically unsound. By its very definition general education implies that part of the educational experience which is good for every one. Then it must be valuable to those able in mathematics just as well as to those who are less apt in this subject.

That is a point which the group carrying on an experimental mathematics program in Southern California[14] have kept in mind. This study is to run for six years (from 1948, it is judged) and should be one to follow closely by every one interested in general education. There are many phases of the California study which are of interest. In the first place the program grew out of a philosophy of education which emphasized the needs of all pupils. A second point that is clear from consideration of the study is that the planning included all interested groups. Colleges and universities were consulted concerning waiving of entrance requirements and to assist in the college prepara-

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[12] Hawkins, G. E., "Adjusting the Program in Mathematics to the Needs of Pupils," *The Mathematics Teacher*, XXXIX: 210, May 1946.

[13] *Loc. cit.*, p. 163.

[14] Carpenter, Dale and Fabing, Charles, "The Experimental Mathematics Program," *California Journal of Secondary Education*, 23: 429–432, November 1948; or Carpenter, Dale, "Planning a Secondary Mathematics Curriculum to Meet the Needs of All Students," *The Mathematics Teacher*, XLII: 41–48, January 1949.

tory series of mathematics courses; parents were acquainted with the proposed changes and were asked for suggestions *before* it was put into practice; pupils had their say, too, being given an opportunity to understand the choices open to them between the new courses and the traditional algebra and geometry; last of all, but most important, principals and teachers sat down together to work out details of the experimental program. This was very important because planning which is perfect on paper sometimes presents insuperable administrative difficulties.

There is one last comment that needs to be made on this study. Notice the names of the courses: Basic Mathematics I, II, III, IV; Industrial Mathematics I, II, III, IV, V, VI; Mathematics I, II, III, IV. All of these are one semester courses. Please observe there is no algebra, no geometry, no arithmetic mentioned as such. It is only mathematics (to which no stigma attaches) which is named. At one stroke (plus a great deal of careful thought, many meetings, and democratically arrived at decisions, I presume) the California committee started anew.

Experiments of this sort ought to be organized elsewhere in the country. It is this forward looking planning of a mathematics program for a local environment and for a local philosophy of education which will "save" the subject of mathematics for boys and girls. And if mathematics is saved for boys and girls then it will find its rightful and deserved place in general education.

DR. POTZGER GIVEN HOLCOMB AWARD

Dr. John E. Potzger, professor of botany, has been named recipient of the J. I. Holcomb award for the past school year. The award is given to the faculty or staff member making the most significant contribution to the welfare and progress of the university during the academic year.

The citation for the award was read by President M. O. Ross during the 94th annual commencement exercises. "As a scientist, Dr. Potzger has, in the midst of full teaching and academic duties, quietly, diligently and authoritatively carried on research, becoming a critical collector and student of grasses, and international authority in the field of forest ecology and in the field of pollen analysis in relation to forest migration, and a recognized leader in the field of science education.

"He has been the holder of research grants on repeated occasions, both from the Indiana Academy of Science and from the American Philosophical Society; and an active participant, through the presentation of scientific papers, and holding office in scientific societies.

"His accomplishments as a teacher, inspirer of students, and contributor to science have been outstanding."—*The Butler Collegian*.

# SOME CLASSROOM NOTES ON IMPACT AND MOMENTUM

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The subject of collision processes in mechanics permits the classroom treatment of a number of topics which possess the unique property of inciting the student's interest. This article suggests a few investigations which the writer has found valuable instructionally. The reader will doubtless recognize some as the usual ones found in textbooks, but it is hoped that several will arouse new interest. Obviously, to treat the topics thoroughly would require a textbook. It will suffice here to state the problem and suggest the results.

Two Distinct Effects of a Force

When a force acts upon a body two very different mechanical effects are produced. The one, called *work*, we evaluate by $\int_0^s F\, ds$; the other, called *impulse*, we measure by $\int_0^t F\, dt$. (Similarly, the dynamic result of a torque may be either work or an angular impulse.) If, in the last expression, we utilize Newton's Second Law and replace F by mv, the integral yields the familiar vector quantity mv, called the momentum. It is valuable to point out that Newton stated his law in precisely this fashion, to wit, $F = \dfrac{d}{dt}(mv)$, where F is the resultant of *all* the forces acting upon the mass m. Since *internal* forces come in pairs and mutually cancel, F must embrace only the external forces. And if the resultant external force is zero, $d(\sum mv)/dt = 0$, hence $\sum mv = $ const. This is the principle of conservation of momentum, and is monumental. It is important to recognize also that while impulse and momentum are vector quantities, work, or energy, is not.
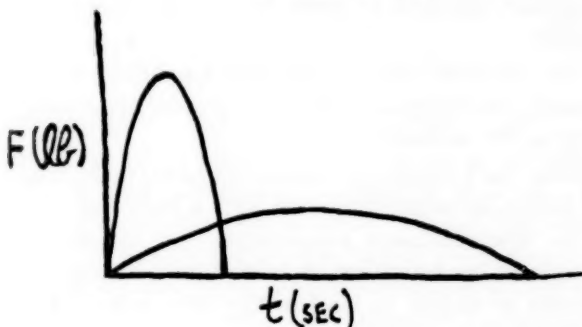
A further detail is instructive at this point. As a rule, neither F nor t are separately evaluated in $\int_0^t F\, dt$ but the value of the integral can

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be established, which is what one wishes anyway. This is interestingly shown graphically by plotting two impulses on the same scale, partitioning the F and t arbitrarily. Thus, the two impulses shown are identical in value, since the areas bounded are equal, but the partitions of F and t are very different.

We suggest at this point a few simple investigations for the student to pursue.

1. A loaded gun at rest with barrel horizontal, shoots a projectile. The forward momentum of the bullet $m_s V$ is equal to the recoil momentum of the gun $M_g v$. The total momentum is zero, both before firing and after. But what about the kinetic energy? This was obviously zero *before* firing and is certainly not zero *after* firing. Also, which has the larger energy, the gun or the bullet? Which, then, is "harder" to stop?

2. A flatcar is at rest. A man stands at one end and fires a bullet into a solid target rigidly fixed to the car at the other end. What happens?

3. A body is at rest. It suddenly explodes and flies to pieces. The separate pieces are suddenly connected. Will any of them continue to move?

4. Could we make a gun with no recoil? Certainly. Have the barrel open at both ends, the firing mechanism in the middle. Now fire identical charges and bullets in both directions!

5. Examine the momentum process when a ball is thrown upward from the earth. It slows down, stops, returns, hits the earth. Is there conservation of momentum?

6. A diver leaps from a springboard. He twists and turns in the air. Can he alter the path of his center of gravity?

7. A man stands on perfectly smooth ice. Prove analytically that the best he can do is to move his center of gravity in a vertical line only.

8. A large spherical balloon supports a gondola which is airtight. How may the balloonist cause the gondola to rotate if he has no access to the outside?

9. Consider the ballistic pendulum. A penetrable mass M (wood, say) hangs on string supports and a bullet of mass m is fired into it. The mistaken assumption (not uncommon) that the principle of conservation of *energy* may be applied to the impact can be shown to lead to the very interesting result that $m = M + m$.

10. A shell flies in a parabolic path. Let it explode somewhere in midair. Suppose one half flies off at right angles to its instanta-

neous path. What does the other half do? What happens to the center of gravity of the system?

COMPRESSION AND RESTITUTION

The *central* collision of two bodies—that is, collision along their line of centers—is easily treated. The usual momentum equation is written, with proper regard for algebraic sign. If restitution is involved it is usually defined in terms of relative velocities before and after impact. But it is instructive to point out that the collision process consists of two parts. Before impact the centers approach; after impact they recede. Hence the separation of the centers must at some time have been minimum. This instant we might call the *instant of greatest compression*. In general, the instant at which the relative velocity along the line of centers vanishes is the instant of greatest compression. Obviously, motion of approach changes to motion of recession. Since the time of action of the forces producing this change is extremely short, the forces are considered *impulsive*. We may accordingly speak of the *impulse of compression* and of the *impulse of restitution*. The ratio of these impulses, which depends upon the *nature* of the masses, is commonly called the coefficient of restitution or coefficient of elasticity, *e*. Mathematically, if I_c is the impulse of compression, and I_r the impulse of restitution, $e = I_r/I_c$, which measures this resilient property. It is suggested that coefficient of resilience is a preferable name. For ideal nonresilience this coefficient is zero. For ideally resilient bodies $e = 1$. Obviously *e* cannot exceed unity, for if it did the restitution process would yield greater energy than that absorbed in the compression process!

DIRECT IMPACT OF A MASS ON A SMOOTH SURFACE

When a mass impinges on a fixed surface, the impact may be direct or oblique, and the surface smooth or rough. For a mass *m*, velocity at impact *v*, normal to a smooth surface, no tangential impulse arises since no sliding occurs, and the recoil velocity is *ev*, along the incident path.

OBLIQUE IMPACT OF A MASS ON A SMOOTH SURFACE

For oblique impact on a smooth surface, no tangential forces exist. If θ is the angle of incidence and ϕ the angle of reflection, it is easily shown that $\tan \theta/\tan \phi = e$. This result is interesting to investigate. When $e = 1$, $\theta = \phi$. When $0 < e < 1$, $\theta < \phi$. When $e = 0$, $\phi = \pi/2$—that is, the mass slides along the plane. This is obviously so, for I_r must be zero. If the kinetic energy process is investigated it can be shown that the loss is given by $E_{lost} = \frac{1}{2}mv^2(1 - e^2)$, where *v* is the normal component of the velocity. This becomes zero if $e = 1$, that is, if the bodies are perfectly resilient. Hence *e* cannot exceed unity.

Oblique Impact of a Mass on a Rough Surface

For oblique impact on a rough surface tangential forces do arise and if μ is the coefficient of friction it may be shown that $e \tan \phi = \tan \theta - \mu(1+e)$. Now $(1+e)\mu$ is always positive. Hence ϕ is less than if the plane were smooth. The results are to be expected. When $\mu = 0$ we have the case already cited. When $\mu = \infty$, $\phi = \pi/2$, which has interesting physical significance. What does the impacting body do? When $e = 0$ what happens?

Central Collision of Two Bodies

If masses m_1 and m_2 have velocities u_1 and u_2 before collision and velocities v_1 and v_2 after impact, it is interesting to show that $u_1 - u_2 = I_c(1/m_1 + 1/m_2)$ and $v_1 - v_2 = -I_r(1/m_1 + 1/m_2)$; then

$$e = \frac{I_r}{I_c} = \frac{v_2 - v_1}{u_1 - u_2},$$

which is the usual "definition" of e. Again, if the kinetic energy process is investigated the loss can be shown to be $(1 - e^2)$ times the original kinetic energy. That is,

$$E_{\text{lost}} = (1 - e^2)(\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2),$$

which becomes zero if $e = 1$. Further, if $e = 0$, all is lost! The results confirm prediction. It may also be stated that the wasted energy is proportional to the square of the relative velocity of approach. This is best revealed by showing that

$$E_{\text{lost}} = \frac{m_1m_2}{2(m_1 + m_2)} (u_1 \pm u_2)^2(1 - e^2)$$

Here $(u_1 - u_2)$ is the relative velocity of approach if both masses are moving in the same sense, and $(u_1 + u_2)$ is the relative velocity if the senses of the velocities are opposite.

If the bodies have ideal recovery of shape and original dimensions and ideal restitution of the energy expended during the deformation, the equations for conservation of momentum and of energy lead to $u_1 - u_2 = v_2 - v_2$, whence $e = 1$, as is expected. Moreover, if $m_1 = m_2$, we find that $u_1 = v_2$ and $u_2 = v_1$, and conclude that in the direct collision of these ideally resilient and equal masses an exact interchange of velocities takes place.

All that has been said here still holds for collisions which are not central. If the collision is oblique, one need only take the components of the velocities along the line of centers at the instant of collision. Also, if the bodies have motion other than pure translation what we have said still holds for the centers of mass provided the collision is

central. For spinning bodies the impact, in general, is not central, and for such eccentric impact (eccentricity due to spin or geometry), considerations of angular momentum must be introduced.

THE PILE DRIVER

When the impact is central and one body is at rest, interesting practical cases arise. It is frequently not realized that a blow or impact may produce two distinctly different mechanical effects. The impact of the hammer on a nail, say, or of the pile driver on the pile, is intended to deform very little and drive very far. On the other hand, in shaping a metal or heading-over a rivet, or in forging in general, the object is to deform as much as possible and drive very little. In the case of the pile driver it is not difficult to show that the proportional loss of kinetic energy (obtained by dividing the loss by the original energy) is given by $(1-e^2)/(1+m/M)$ whence, the loss is less when m (the mass of the driver) is large compared with M (the mass of the pile). Is it not easier to drive a stake into the ground with a heavy hammer than with a light one? And this is so even if the light one has a high enough velocity that its kinetic energy is the same as that of the slow-moving heavy hammer!

If we refer to the expression for the energy lost, namely

$$E_{\text{lost}} = \tfrac{1}{2}(1-e^2)\left(\frac{m_1 m_2}{m_1+m_2}\right)(u_1-u_2)^2,$$

we again see that it is advantageous to make the driver heavy compared to the pile. It is also advantageous to make e large! What happens when $e=1$? Remember that our purpose here is to transform the kinetic energy of the hammer into kinetic energy of the pile which in turn overcomes the ground friction.

An excellent extension of the pile-driver problem is the following:
1. If the driver of mass m is at a height h above the pile of mass M, and if on impact the pile moves down a distance d as a consequence of the blow, find the dead load which must be applied to the pile so that it will just "run."

For forging, obviously, we wish to dissipate the kinetic energy of the hammer by deforming the object. When the body is hot, e may be very nearly zero. The equation above may then be written

$$E_{\text{lost}} = \tfrac{1}{2}\,\frac{m_1 m_2}{m_1+m_2}\,u_1{}^2$$

or better still

$$E_{\text{lost}} = \tfrac{1}{2} m_1 u_1{}^2\,\frac{m_2}{m_1+m_2}\,.$$

Accordingly, the energy lost is $m_2/(m_1+m_2)$ times the original energy of the hammer. For maximum dissipation, must not this ratio be as close to unity as possible? Whence we conclude that the mass of the anvil m_2 must be as large as possible compared with the hammer as mass m_1. The results are confirmed by experience.

It is instructive to refer to these results in terms of the "efficiency" of the blow or impact for the job to be done. In the pile-driver case the efficiency obviously means

$$\frac{\text{driving energy}}{\text{total energy}}$$

in the forging case, efficiency means

$$\frac{\text{deforming energy}}{\text{total energy}} .$$

This point of view leads to the following:

Driving "efficiency" is $\dfrac{m}{M+m}$:

Deforming "efficiency" is $\dfrac{M}{M+m}$.

Finally, we wish to point out briefly a flagrant misconception which resides in the minds of many. It is to the effect that a heavy-caliber rifle bullet has large "stopping" power. We refer here to the purely mechanical effect of its impact, and make no reference to the physiological aspects. It is not uncommon for big-game hunters and Army men to suggest that such and such a bullet can "knock a man down," or a beast! And descriptive passages pointing to this are to be found. The error in this thinking is obvious; for the forward momentum of the bullet cannot be one wit greater than the backward or recoil momentum of the gun, and hence the bullet can deliver no greater impulse on the body it strikes than the gun can deliver to the marksman!! In "big game" accounts it is often related how the shot "stopped a charging rhinoceros"!!

ROCKET AND JET PROPULSION

We first dismiss the student's erroneous but not uncommon misconception that the rocket needs an atmosphere to operate. The impulses it receives are provided only by its own exhaust gases and accordingly the atmosphere seriously reduces its performance. It is suggested that the student operate a rotary lawn sprinkler in a tub of water!

The rocket of mass M is initially at rest. Its initial momentum is

zero. When its fuel burns, the exhaust gases acquire a momentum and consequently the rocket acquires an equal and opposite momentum— effectively the recoil. Let the rocket now have a velocity V. If a mass m is ejected from its tail with a velocity v (relative to the rocket) the rocket will get a boost; call it ΔV. Then, since momentum is conserved we may write at once $(M - m)\Delta V = mv$, or more exactly $(M - m)(V + \Delta V) - MV = m(v - V)$, which reduces to the same thing.

Energy considerations are now entirely different. At the very start of its flight the rocket is high inefficient. This is because nearly all of the energy developed in the initial stages is used up in imparting kinetic energy to the exhaust gases. Indeed, this is why a rocket is often given a "boost" by some auxiliary device. Consequently little energy is acquired by the rocket itself. However, this state of affairs is happily only transient, for as the rocket gains velocity the exhaust gases leaving the rocket with a certain velocity lose velocity relative to the earth. Thus, when the rocket velocity relative to the earth equals the exhaust velocity relative to the rocket, the exhaust velocity is zero relative to the earth. Consequently the exhaust energy is zero. At this velocity the fuel energy is all taken up by the rocket. The partition of energy may be investigated as follows:

(a) Kinetic energy of exhaust gases is $\frac{1}{2}m(v - V)^2$;

(b) Kinetic energy of rocket after gaining velocity, ΔV is $\frac{1}{2}(M - m)(V + \Delta V)^2$;

(c) Original kinetic energy of rocket is $\frac{1}{2}MV^2$.

The change in kinetic energy, therefore, acquired from the burning of the fuel, is obviously the quantity $a + b - c$. However, only the quantity $b - c$ is gained by the rocket. The quantity a is wasted. It is clear then that when $v = V$ this waste is nil. Briefly, maximum efficiency now occurs. The velocity of the escaping gases is zero relative to the earth, or, saying it another way, the gases are ejected with the same velocity as that possessed by the rocket. We may interpret the ratio $(b - c)(a + b - c)$ as the "efficiency" of the device.

Finally, we put the question: can a rocket go faster than its exhaust?

In jet propulsion these results also hold, but in this mode of flight air is taken in at the front to oxidize the charge in the combustion chamber. Furthermore, gas discharge is at a more or less uniform rate and with a fairly constant velocity. Will *this* mechanism work better above our atmosphere?

Retreat may be success,—
Delay, best speed, half loss, at times, whole gain.
—*Robert Browning.*

THINKING VERSUS DOING IN BIOLOGY*

MAITLAND P. SIMMONS

Irvington High School, Irvington, New Jersey

It is commonly accepted today that the work of the classroom should provoke interest in thinking and doing on the part of the pupil, rather than emphasize facts and principles. Especially this is true for science. Many teachers have failed to make the best possible use of teaching helps for carrying out this objective. It is with this thought in mind that the following specimens and illustrative materials have been selected.

Before the pupil begins this activity, it is desirable that he acquaint himself with the subject by answering the preceding informative questions which should be within his scope.

The activity will consist of making drawings from their specimens and writing answers from their observational questions.

The experience is followed by interpretative questions which should summarize the important thoughts.

UNIT: PLANT LIFE

Topic Study: Flowers

Introduction

1. What is the national flower of the United States? How and when was it selected?

2. Name some plants that have brightly colored blossoms with strong odor, plants that have brightly colored blossoms with little odor, plants that have inconspicuous blossoms with little or no odor.

3. Why are color and odor important characteristics of flowers?

4. Name and identify at least ten cultivated flowers found in your state.

5. Report on the works of Charles Darwin and John Burroughs.

6. Name some common flowers which contain poison in their foliage.

7. Write a report on your visit to a florist shop.

8. Consult a seed catalog for reference, and list the flowers used for food and those used for ornament.

9. Name some flowers used for dyes, medicines, and perfumes.

10. Give an oral report on the work of the man who created the thornless cactus and developed the nectarine. How did he do it?

11. What flowers are attractive to humming birds? Why?
12. Name the kinds of insects you have seen around flowers.

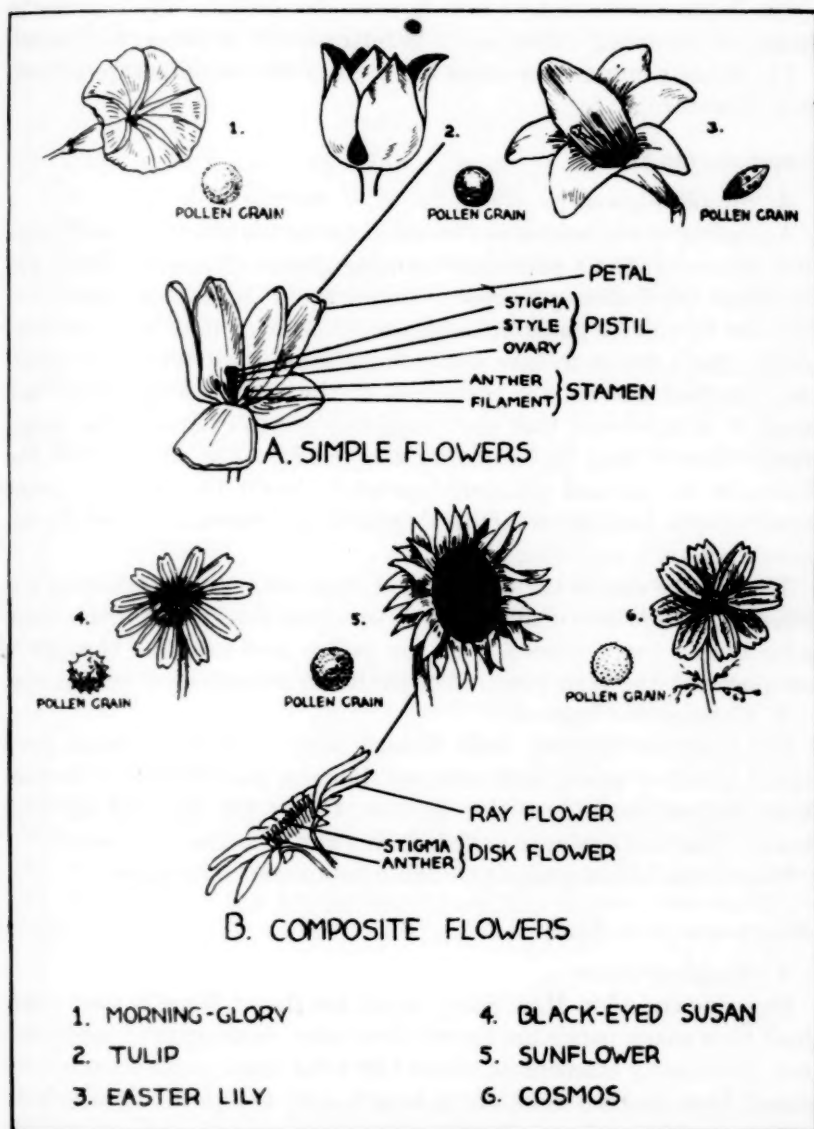ACTIVITY: STRUCTURAL CHARACTERISTICS OF COMPLETE FLOWERS

FIG. 1.—Types of Flowers.

13. Explain four ways of cross-pollination.
14. Name a flower that lacks both sepals and petals in its struc-

ture; a flower which affects human sensibilities more deeply than others.

15. Which are the more desirable flowers to grow in the home-garden, annuals or perennials? Give reasons for your answer.

16. Make a collection of wild flowers. Do not pick any that are in danger of becoming extinct such as bitter-sweet, arbutus, and laurel.

17. What common materials are used in the manufacture of artificial flowers?

Directions for Study:

A. Simple Flowers†

According to the season and location, bring to class common flowers (full bloom) such as petunias, morning-glories (*Heavenly Blue*), nasturtiums, daffodils, primroses, snapdragons, and geraniums. For size and simplicity gladioli, tulips, and Easter or tiger lilies are preferred. Ask a florist to save you faded Easter lilies. Whenever possible, out-door flowers are preferred. If the collected specimens are small, it is important that each pupil has one to examine. The large, showy flowers may be shown by the teacher. For study, hold the flower in its natural position (upright). Smell the flowers, count sepals (green, leaf-like structures), petals, and stamens. Carefully remove each part including pistil.

Toward the end of the class period, dust some pollen grains of the different flowers into drops of water on clean glass slides. Place thin, unbreakable cover glasses over the pollen and examine through a low-powered and high-powered objective of a compound microscope.

B. Composite Flowers†

Use common flowers (full bloom) such as daisies (black-eyed susan), cosmos, asters, and coreopsis. For size and simplicity the sunflower is preferred. Smell the flowers. Count the ray and the disk flowers. Carefully remove a disk flower and examine the parts.

View some pollen grains through a compound microscope.

Observations from Study:

A. Simple Flowers

Describe the odor. How many sepals are there? What is their function? How many petals are there? Give color, arrangement, and function. How many stamens are there? Of what three parts is each composed? How do they compare in length with the pistil? In which direction are the anthers pointing, inside toward the pistil or outside and away from it? Why? What color are the anthers? Why? Describe the pollen grains. Give location, description, and function of pistil.

† This part of the experiment will probably take a 45 minute period to complete

Of what three parts is it composed? How is the stigma adapted for carrying pollen?

B. Composite Flowers

Describe the odor. How many ray and disk flowers does the specimen have? Give location, color, and function of ray and disk flowers. Can you find any pistils and stamens? Give a description of them. Describe the pollen grains.

INTERPRETATIONS

1. Why may the petal of a flower be called the advertisement?

2. What is the advantage of keeping bees in an orchard?

3. Why is the snapdragon especially attractive to insects?

4. Explain why most night-blooming flowers are white.

5. How are flowers adapted for insect pollination? What is the advantage of the sticky material secreted by the stigma?

6. Why do petals of an apple blossom, or in fact any flower, drop off after fertilization takes place?

7. Explain fertilization. What is the result?

8. Which flowers develop more fully, those in a cluster or solitary ones? Explain.

9. What conservation laws have been made in your state to preserve wild flowers?

10. Why is seed conservation especially important during a war? What care should be taken in selecting seeds? Name the largest and the smallest seed.

11. What are the differences between wild and cultivated flowers?

12. Tell how the name Jack-in-the-Pulpit came to be adopted.

13. Why are flowers useful to both plant and human life?

14. What flowers are adapted for growing in window boxes? Why?

15. Name some flowers that are appropriate for making attractive bouquets. Why isn't it a good plan to pick the hardiest flowers?

16. What benefits can be derived by taking care of a flower garden?

17. Explain why one fig tree bears fruit, and another near by bears blossoms but no fruit.

WHAT COLLEGE STUDENTS THINK OF US

My students in secondary science were required to select the science periodical which in their respective opinions was most practical, most readable and in general offered the "grass roots" science teacher the most help. All had an equivalent exposure to all of the available journals in that area. Thirty-one (31) of thirty-four (34) selected SCHOOL SCIENCE AND MATHEMATICS.

From a Teacher in a Leading College

RECOMMENDED REVISION OF THE BY-LAWS

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS

Present Articles and Sections to be Amended

I. MEMBERS

SECTION I. QUALIFICATIONS: Any teacher concerned in the teaching of science or mathematics is eligible for regular membership in this association. Any person, firm, or corporation, interested in any aspect of the teaching of science or mathematics shall be eligible for associate membership in this association.

SECTION IV. DUES: The annual dues of each member of the association shall be $2.50 payable in advance. The annual dues shall also entitle the member to receipt of the Journal or other publication of the association.

SECTION V. VOTING: Each regular member of the association shall be entitled to one vote. Associate members shall be entitled to attend all meetings of the association, but shall not be entitled to vote.

III. OFFICERS

SECTION I. OFFICERS: The officers of this association shall be a President, a Vice President, a Secretary and a Treasurer. One or more Assistant Secretaries and Assistant Treasurers may be appointed by the President.

SECTION II. QUALIFICATIONS: The President and Vice President shall be members of the Board of Directors.

SECTION III. ELECTION, TENURE OF OFFICE, COMPENSATION: The President and Vice President shall be elected by the members of the association at the annual meeting and shall serve for a term of one year or until their successors are elected. All other officers shall be elected by the Board of Directors at a meeting to be held following the annual meeting of the association and shall serve for a term of one year or until their successors are elected. The compensation of the officers, if any, shall be fixed by the Board of Directors.

Suggested Revisions (Deletions are bracketed; additions are italicized)

I. MEMBERS

SECTION I. QUALIFICATIONS: [Any teacher concerned in the teaching of science or mathematics is eligible for regular membership in this association.] Any *teacher or other* person, firm, or corporation, interested in any aspect of the teaching of science or mathematics shall be eligible for [associate] membership in this association.

SECTION IV. DUES: The annual dues of each member of the association shall be [$2.50] *$3.50*, payable in advance. The annual dues shall [also] entitle the member to receipt of the Journal [or other publication of the association.]

SECTION V. VOTING. Each [regular] member of the association shall be entitled to one vote. [Associate members shall be entitled to attend all meetings of the association, but shall not be entitled to vote.]

III. OFFICERS

SECTION I. OFFICERS: The officers of this association shall be a President, *a President-Elect*, a Vice President, a Secretary, a Treasurer *and Business Manager, an Editor of the Journal, and an Historian.* One or more Assistant Secretaries and Assistant Treasurers may be appointed by the President.

SECTION II. QUALIFICATIONS: The President, *the President-Elect*, and the Vice-President shall be [members of] *nominated from* the Board of Directors.

SECTION III. ELECTION, TENURE OF OFFICE, COMPENSATION: The *President, President-Elect,* and Vice President shall be elected by the members of the association at the annual meeting and shall serve for a term of one year or until their successors are elected. *The Treasurer and Business Manager, Editor of the Journal, Historian, and Secretary shall be elected by the Board of Directors at a meeting to be held following the annual meeting of the association, and shall serve for a term of three years.*

Their terms may be renewable. All other officers shall be elected by the Board of Directors at a meeting to be held following the annual meeting of the association, and shall serve for a term of one year or until their successors are elected. The compensation of the officers, if any, shall be fixed by the Board of Directors.

SECTION IV. POWERS AND DUTIES OF OFFICERS: (a) PRESIDENT: The President shall preside at all general meetings of the association and shall perform the usual duties of his office. He shall be chairman of the Board of Directors and chairman of the Executive Committee, and shall perform the usual duties of those offices.

SECTION IV. POWERS AND DUTIES OF OFFICERS: (a) PRESIDENT: The President shall preside at all general meetings of the association and shall perform the usual duties of his office. He shall be chairman of the Board of Directors and chairman of the Executive Committee, and shall perform the usual duties of those offices.

(b) *President-Elect: The President-Elect shall be considered as the su :cessor to the Presidency. He shall act for the President in the latter's absence. He shall be a member of the Board of Directors and Executive Committee and shall work closely with the President to familiarize himself with matters of policy and procedure.*

(b) VICE PRESIDENT: He shall act for the President in the latter's absence.

(c) VICE PRESIDENT: He shall act for the President [in the latter's absence.] *or President-Elect in event of the absence of either. He shall also be a member of the Executive Committee.*

(c) SECRETARY: The Secretary shall keep all records, minutes of all meetings and shall prepare and submit a complete report of the annual meeting to the Editor of the Journal by December 31st following the meeting.

(d) SECRETARY: The Secretary shall keep all records, minutes of all meetings and shall prepare and submit a complete report of the annual meeting to the Editor of the Journal by December 31st following the meeting.

(d) TREASURER: The Treasurer shall collect all dues and hold all moneys and keep a record of all receipts and disbursements. He shall give a detailed report at each meeting of the association. He shall pay out funds on the order of the Board of Directors and the Executive Committee.

(e) TREASURER *and Business Manager:* The Treasurer *and Business Manager* shall collect all dues and hold all moneys and keep a record of all receipts and disbursements. He shall give a detailed report at each meeting of the association. He shall pay out funds on the order of the Board of Directors and the Executive Committee. *He shall also act as Business Manager of the Journal, in cooperation with the Editor.*

(e) EDITOR AND BUSINESS MANAGER: The Editor and Business Manager of the Journal shall be elected by the Board of Directors. Their term of office shall be indeterminate at the

(f) EDITOR [AND BUSINESS MANAGER] *of the Journal:* The editor [and the Business Manager] of the Journal shall be [elected by the Board of Directors] *responsible for the Journal.* [Their

*Present Articles and Sections
to be Amended*

Suggested Revisions (Deletions are bracketed; additions are italicized)

pleasure of the Board of Directors.

term of office shall be indeterminate at the pleasure of the Board of Directors.]

(g) *Historian: The Historian shall be charged with the responsibility of collecting and preserving the historical documents of the association.*

IV. BOARD OF DIRECTORS

SECTION II. NUMBER: There shall be fifteen (15) members of the Board of Directors. The President, the Vice President, and the President of the preceding year shall be ex-officio members of the Board of Directors. The remaining members of the Board of Directors shall be divided into three groups of four (4) directors each. Four directors shall be elected annually to succeed those of the group whose terms are about to expire.

IV. BOARD OF DIRECTORS

SECTION II. NUMBER: There shall be [fifteen (15)] *twenty (20)* members of the Board of Directors. The President, the Vice President, the President of the preceding year, *the President-Elect, the Secretary, the Treasurer and Business Manager, the Editor of the Journal, the Historian* shall be ex-officio members of the Board of Directors. The remaining members of the Board of Directors shall be divided into three groups of four (4) directors each. Four directors shall be elected annually to succeed those of the group whose terms are about to expire.

SECTION IV. ELECTION, TENURE OF OFFICE AND COMPENSATION: Directors shall be elected by a majority vote of the members present at any annual meeting. They shall assume the duties of their office immediately preceding the adjournment of the annual meeting and shall serve for a period of three years or until their successors are elected. They shall serve without compensation, except that they may be allowed a reasonable compensation for traveling, and necessary expenses incurred by them in the discharge of their official duties.

SECTION IV. ELECTION, TENURE OF OFFICE AND COMPENSATION: Directors shall be elected by a majority vote of the members present at any annual meeting. They shall assume the duties of their office immediately preceding the adjournment of the annual meeting and shall serve for a period of three years or until their successors are elected. They shall serve without compensation, except that they may be allowed a reasonable compensation for traveling, and necessary expenses incurred by them in the discharge of their official duties.

Vacancies in the Board of Directors may be filled by the Board of Directors at any meeting thereof. The directors so chosen shall serve for the unexpired term of the director whom he succeeded.

Vacancies in the Board of Directors [may] *shall* be filled by the Board of Directors at any meeting thereof. [The directors] *A director* so chosen shall serve for the unexpired term of the director whom he succeeded.

Whenever directors are elected, whether at the expiration of a term or to fill vacancies, a certificate under the seal of the association giving the names of those elected and the terms of their office shall be recorded in the office of the recorder of deeds where the certificate of organization is recorded.

Whenever directors are elected, whether at the expiration of a term or to fill vacancies, a certificate under the seal of the association giving the names of those elected and the term of their office shall be recorded *by the Treasurer and Business Manager* in the office of the recorded of deeds where the certificate of organization is recorded.

SECTION VIII. POWERS AND DUTIES: The Board of Directors shall (1) have general supervision of the activities of

SECTION VIII. POWERS AND DUTIES: *Members of* the Board of Directors shall (1) *have the right to vote;* (2) have

*Present Articles and Sections
to be Amended*

the association; (2) authorize the expenditure of funds; (3) fix the salary and bonds of the officers; (4) fill vacancies.

V. EXECUTIVE COMMITTEE AND
OTHER COMMITTEES

SECTION I. EXECUTIVE COMMITTEE: (a) MEMBERS: Members of the Executive Committee shall consist of the President and two directors chosen by the Board of Directors, and shall serve for one year or until their successors have been elected or appointed.

SECTION IV. PROFESSIONAL SECTIONS: The association shall be divided into sections as follows: Biology, Chemistry, Elementary Science, General Science, Geography, Mathematics, and Physics. Each section shall be composed of members of the association who are especially interested in the subject of that section. The organization and activities of the sections may be amended from time to time by the Board of Directors. Unless otherwise provided by the Board of Directors each section shall elect its own Chairman, vice Chairman and Secretary.

Suggested Revisions (Deletions are bracketed; additions are italicized)

general supervision of the activities of the association; (3) authorize the expenditure of funds; (4) fix the salary and bonds of the officers; (5) fill vacancies.

V. EXECUTIVE COMMITTEE AND
OTHER COMMITTEES

SECTION I. EXECUTIVE COMMITTEE: (a) MEMBERS: Members of the Executive Committee shall consist of the President, *President-Elect, Vice President, and* immediate past president [and two directors chosen by the Board of Directors] and shall serve for one year or until their successors have been elected or appointed.

SECTION IV. PROFESSIONAL SECTIONS *and Groups*: The association [shall] *may* be [divided] *organized* into sections *and groups* as follows: Biology, Chemistry, *Elementary Mathematics*, Elementary Science, General Science, Geography, Mathematics, Physics, *Conservation, Junior College Group, Senior High School Group, Junior High School Group, and Elementary School Group*. Each section *or group* shall be composed of members of the association who are especially interested in the subject of that section *or group*. The organization and activities of the sections *or groups* may be amended from time to time by the Board of Directors. *Sections or groups may be added, combined, or discontinued by the Board of Directors, as the demand indicates.* Unless otherwise provided by the Board of Directors each section *or group* shall elect its own Chairman, vice Chairman and Secretary.

Section VII. The Policy and Resolutions Committee: The Policy and Resolutions Committee shall consist of six (6) members. Each member shall serve three years. Two members shall be chosen annually by the President to replace the two whose terms are about to expire. One of those chosen annually must be from the Board of Directors. The President will annually designate the chairman of this committee.

AMENDMENT X. Article 6. Amendments. The By-Laws may be amended by a two-thirds (⅔) vote of the regular members present at any regular meeting of the Association, or at any special meeting of the Association called for the

purpose of voting on such amendment provided the proposed amendment has been published in two successive issues of SCHOOL SCIENCE AND MATHEMATICS.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON
State Teachers College, Kirksville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution as on the following pages.

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you suppose if you have one and also the source and any known refernces to it.

3. In general when several solutions are correct, the ones submitted in the best form will be used.

Late Solutions

2125, 6, 8. *J. de Boer, Leiden, Holland.*

2126, 34, 5, 6. *Charles M. Warden, Warrensburgh, Missouri.*

2125. *Steve Dormen, Indiana, Pennsylvania.*

2131, 4, 5, 6, 7, 9. *Alan Wayne, Flushing, Long Island, New York.*

2135. *Warren Hulser, Hebron, Maine.*

2134, 6. *Winfield M. Sides, Andover, Mass.*

2131, 2, 4, 5, 6. *W. J. Blundon, St. John's, Nfld.*

2137. *Proposed by V. C. Bailey, Evansville, Indiana.*

If a point be taken within an equilateral triangle, such that its distances from the vertices are equal to the sides a, b, c of another triangle, show that the angle between these distances will be

$$60° + A, \qquad\qquad 60° + B, \qquad\qquad 60° + C.$$
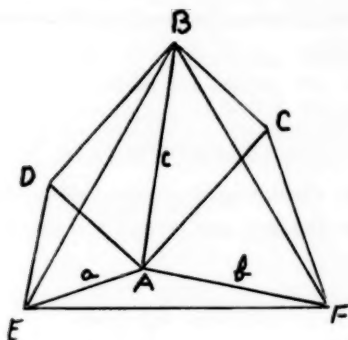
Solution by W. J. Cherry, Berwyn, Ill.

Let A be any print in equilateral triangle EFB. Designate the distances AE, AF and AB by a, b and c. With AF and AE as bases, respectively, draw equilateral triangles AFC and AED. Draw DB and CB. Angle $CFB = 60° -$ angle

AFB. Angle $AFE = 60° -$ angle *AFB*. Hence $CFB = AFE$. Then triangles *CFB* and *AFE* are congruent.

Angle $DEB = 60° -$ angle *AEB*. Angle $AEF = 60° -$ angle *AEB*. Hence *DEB* $= AFE$. Then triangles *DEB* and *AFE* are congruent; and consequently triangles *CFB* and *DEB* are congruent.

$DB = b$, and $CB = a$. Then triangles *ABC* and *ADC*, having sides equal to a, b and c, meet the conditions for the second triangle of our hypothesis.

Angle $BAF = 60° +$ angle *BAC*.

Angle $BAE = 60° +$ angle *BAD*.

Angle $EAF = 60° +$ angle *BCA*, since angle $EAF =$ angle *BCF*. This completes the proof.

Solutions were also offered by W. J. Cherry, Berwyn, Ill.; S. E. Field, Ironwood, Mich.; W. R. Talbot, Jefferson City, Mo.; Max Beberman, Shanks Village, N. Y.; C. W. Trigg, Los Angeles; W. J. Blundon, St. John's, Nfld.; and by Proposer.

2138. *Proposed by Norman Anning, University of Michigan.*

In the equation $2x^3 + 2y^3 - 3x^2 - 3y^2 + 1 = 0$, if any rational value be assigned to x, show that at least one rational value can be computed for y.

<div align="center">

First Solution

W. J. Cherry, Berwyn, Illinois

</div>

$$2y^3 - 3y^2 + 2x^3 - 3x^2 + 1 = 0.$$
$$f(y) = 2y^3 - 3y^2 + (1-x)(1-x)(2x+1) = 0.$$

By synthetic division:

2	-3	0	$(1-x)(1-x)(2x+1)$	$\lfloor 1-x$
	$2-2x$		$-(1-x)(1-x)(2x+1)$	
2	$-(2x+1)$	$-(2x+1)(1-x)$	0	

Therefore $1 - x$ is a root of $f(y) = 0$, and any rational value of x gives at least one rational value for y.

<div align="center">

Second Solution

W. R. Talbot, Jefferson City, Mo.

</div>

For any value of x, it is possible to compute y by the usual methods for solving cubics algebraically. If we give x a rational value, the equation becomes a cubic in y with coefficients that are both real and rational. We know, however, that complex roots occur in conjugate pairs if the coefficients are real, and irrational roots occur in conjugate pairs if the coefficients are rational; consequently, this cubic has at least one rational value for y for any rational x.

Solutions were also offered by Max Beberman, Shanks Village, N. Y.; Emery Crane, Mt. Pleasant, Iowa; C. W. Trigg, Los Angeles; and by Proposer.

2139. *Proposed by Norman Anning, University of Michigan.*

Solve the set of equations:

$$x^2 + xy + y^2 = z^2$$
$$x + z = 2y$$
$$(x+3)^2 + (y+3)^2 = (z+3)^2.$$

Solution by S. E. Field, Gogebic Junior College, Ironwood, Mich.

Eliminating x between the first and second and the second and third of these equations, we obtain

$$y(7y - 5z) = 0$$
$$(y+3)(5y - 4z + 3) = 0.$$

Three pairs chosen from these four factors together with the second of the given equations yield solutions:

$y = 0$	$y + 3 = 0$	$7y - 5z = 0$
$5y - 4z + 3 = 0$	$7y - 5z = 0$	$5y - 4z + 3 = 0$
$x + z = 2y$	$x + z = 2y$	$x + z = 2y$

The solutions are:

$x = -3/4$	$x = -9/5$	$x = 3$
$y = 0$	$y = -3$	$y = 5$
$z = 3/4$	$z = -21/5$	$z = 7$

The fourth pair, $y = 0$, $y + 3 = 0$, are incompatible.

Solutions were also offered by C. W. Trigg, Los Angeles; V. C. Bailey, Evansville, Ind.; Warren O. Hulser, Hebron, Maine; J. W. Lindsey, Amarillo, Texas; Felix John, Philadelphia; W. R. Talbot, Jefferson City, Mo.; A. MacNeish, Chicago; Max Beberman, Shanks Village, N. Y.; M. Philbrick Bridges, West Roxbury, Mass.; Emery Crane, Mt. Pleasant, Iowa; Margaret Jouph, Milwaukee, Wisc.; P. S. Marthakis, Salt Lake City; C. McIlroy, Montreal; David Freeman, Mill Valley, Calif.; Donald Krezek, Berwyn, Ill.; Fern S. Gipe, Urbana, Ill.; W. J. Blundon, St. John's, Nfld.; Francis L. Miksa, Aurora, Ill.; and Proposer.

2140. *Proposed by Felix John, Ammendale, Md.*

Find a simple value for

$$\sqrt[3]{\frac{1 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 8 + 3 \cdot 6 \cdot 12 + \cdots}{1 \cdot 3 \cdot 9 + 2 \cdot 6 \cdot 18 + 3 \cdot 9 \cdot 27 + \cdots}}$$

First Solution
Max Beberman, Shanks Village, N. Y.

$$\sqrt[3]{\frac{1 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 8 + 3 \cdot 6 \cdot 12 + \cdots}{1 \cdot 3 \cdot 9 + 2 \cdot 6 \cdot 18 + 3 \cdot 9 \cdot 27 + \cdots}} = \sqrt[3]{\frac{1 \cdot 2 \cdot 4(1^3 + 2^3 + 3^3 + \cdots)}{1 \cdot 3 \cdot 9(1^3 + 2^3 + 3^3 + \cdots)}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}.$$

Second Solution
W. J. Cherry, Berwyn, Illinois

$$\frac{1 \cdot 2 \cdot 4}{1 \cdot 3 \cdot 9} = \frac{2 \cdot 4 \cdot 8}{2 \cdot 6 \cdot 18} = \frac{3 \cdot 6 \cdot 12}{3 \cdot 9 \cdot 27} = \cdots = \frac{n \cdot 2n \cdot 4n}{n \cdot 3n \cdot 9n}$$

$$= \frac{1\cdot2\cdot4+2\cdot4\cdot8\ +3\cdot6\cdot12+\ \cdots\ +n\cdot2n\cdot4n}{1\cdot3\cdot9+2\cdot6\cdot18+3\cdot9\cdot27+\ \cdots\ +n\cdot3n\cdot9n}.$$

The value of the equal ratios is 8/27, and hence the simple value required is $\frac{2}{3}$.

Solutions were also proposed by P. S. Marthakis, Salt Lake City; Charles M. Warden, Warrensburg, Mo.; Gerald Sabin, Tampa, Fla.; C. W. Trigg, Los Angeles; V. C. Bailey, Evansville, Ind.; Malcolm Stuart, Evansville, Ind.; Warren O. Hulser, Hebron, Me.; George Grosman, New York; W. R. Talbot, Jefferson City, Mo.; S. E. Field, Ironwood, Mich.; W. E. Shindle, Houghton, N. Y.

2141. *Proposed by Norma Sleight, Winnetka, Ill.*

Construct a triangle, given base a, the vertex angle, A, and the sum of the other sides, $b+c$.

Solution by A. MacNeish, Chicago

Given: lines a and $b+c$, and angle A.
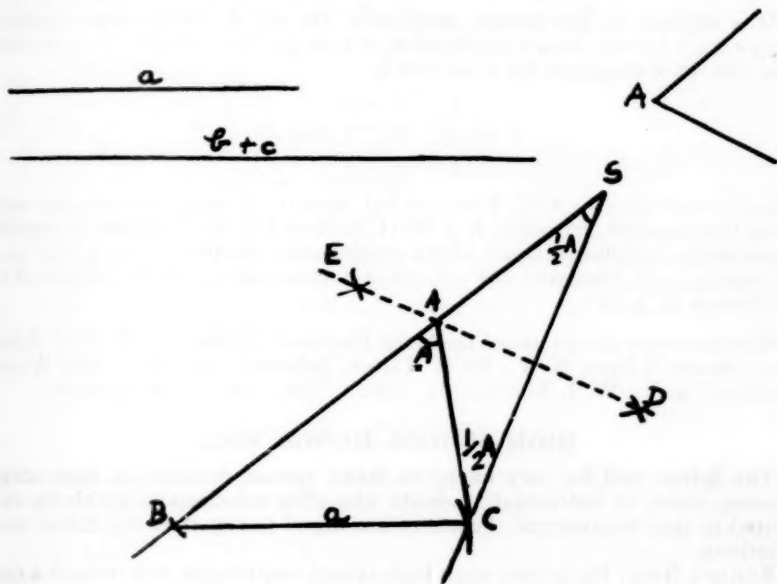
1. Construct an angle equal to $\frac{1}{2}A$, namely $\angle S$.
2. Lay off distance SC equal to line $b+c$.
3. With B as a center and radius equal to a strike an arc cutting the other side of $\angle S$ at C.
4. Construct DE the $\perp$ bisector of SC, intersecting SB at A.
5. Draw AC.
6. Triangle ABC is the required triangle.

Proof

1. The base BC equals the given line a by step 3.
2. $AC = AS$ since A is on the $\perp$ bisector of SC, so $AB + AC = SB$.
3. $\angle ACS = \frac{1}{2}A$, since $\triangle ACS$ is isosceles and they are base $\angle$s.
4. $\angle BAC$ = the given $\angle A$ since an exterior angle of a tirangle equals the sum of the two opposite interior angles.
5. So $\triangle ABC$ has its base = given line a, its vertex $\angle$ = the given $\angle A$, and the sum of its other two sides = the given line $b+c$.

Note. If $a < (b+c)$ sine $\frac{1}{2}A$, the triangle is impossible.

If $a > (b+c)$, the triangle is impossible.

If $a > (b+c)$ sine $\frac{1}{2}A$, but $a < (b+c)$, then the triangle is possible and can be constructed as shown.

Solutions were also proposed by Max Beberman, Shanks Village, N. Y.; Steve R. Domen, Indiana, Pa.; Easley Blackwood, Indianapolis, Ind.; Margaret Joseph, Milwaukee, Wis.; W. R. Smith, Sutters Bay, Mich.; M. Philbrick Bridgess, West Roxbury, Mass.; V. H. Paquet, Milwaukee, Ore.; Charles M. Warden, Warrensburg, Mo.; Betty Stubblefield, Chicago; W. R. Talbot, Jefferson City, Mo.; C. W. Trigg, Los Angeles; S. E. Field, Ironwood, Mich.; W. J. Cherry, Ill.; Fern S. Gipe, Urbana, Ill.; and by the proposer.

2142. *Proposed by C. W. Trigg, Los Angeles City College.*

There is only one set of three distinct positive integers, having no common divisor, greater than unity, such that each is a divisor of the sum of the other two.

Solution by the Proposer

It is required to find integer solutions of the set of simultaneous equations $x+y = mz$, $y+z = nx$, $z+x = py$, where m, n, p are positive integers. The condition that this set of equations has a solution is

$$\begin{vmatrix} 1 & 1 & -m \\ -n & 1 & 1 \\ 1 & -p & 1 \end{vmatrix} = 0.$$

That is, $mnp = m+n+p+2$. There are but three triads of positive integers satisfying this equation, namely: $(1, 2, 5)$, $(1, 3, 3)$ and $(2, 2, 2)$. These correspond, respectively, to three solutions of the simultaneous equations, $(2y, y, 3y)$, $(y, y, 2y)$ and (y, y, y). Therefore, the only set of integers meeting the conditions of the problem is $(2, 1, 3)$.

Solutions were also proposed by Aaron Buchman, Buffalo, N. Y.; Max Beberman, Shanks Village, N. Y.; W. R. Talbot, Jefferson City, Mo.; Alan Wayne, Flushing, N. Y.; W. J. Blundon, St. John's, Nfld.; and by the proposer.

HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each high school contributor will receive a copy of the magazine in which the student's name appears.

For this issue the Honor Roll appers below.

2137, 9, 40, 1. *Phillip Zeidenberg, Brooklyn.*

2126. *Jack Preston Birmingham, Ala.*

2139. *Patricia Henry and Hwa-Chang Lu, Convent Station, N. J.; Melvin Novick, Chicago; Robert Schaefer, Griffin, Ohio; Mike Kennan and Edward Mueche, Portland, Ore.*

2141. *Rita Dudak and Laura Marie Hegge, Convent Station, N. J.; Robert Runyon, Arlington Heights, Ill.*

PROBLEMS FOR SOLUTION

2155. *Proposed by Francis L. Miksa, Aurora, Ill.*

Show that if two concentric ellipses are tangent to each other, the angle between their major axes is

$$\text{arc tan} \sqrt{\frac{(c^2-a^2)(d^2-b^2)}{(c^2-b^2)(a^2-d^2)}} \quad \text{where } a, b \text{ and } c, d \text{ are the semi-axes.}$$

2156. *Proposed by W. R. Talbot, Jefferson City, Mo.*

Find the form of the roots of the cubic equation $f(x) = 0$ for which the two areas enclosed by $y = f(x)$ and the x-axis are integers.

2157. *Proposed by Olive Ireland, St. Albans, Vt.*

If in triangle ABC, $\angle A = 2 \angle B$, then show that $a^2 = b(c+b)$.

2158. *Proposed by V. C. Bailey, Evansville, Ind.*

Show that the length of the side of the least equilateral triangle that can be drawn with its vertices on the sidex of a given triangle ABC is

$$\frac{2\Delta\sqrt{2}}{\sqrt{a^2+b^2+c^2+4\sqrt{3}\cdot\Delta}}$$

where Δ is the area of ABC.

2159. *Proposed by C. W. Trigg, Los Angeles City College.*

If $f(x) = x^3 + 10x^2 + mx - 12 = 0$ and $g(x) = x^3 + 2x^2 + nx + 2 = 0$, have two roots in common, solve each equation completely.

2160. *Proposed by W. H. Cleveland, Meridian, Miss.*

Three bells commenced to toll at the same time, and tolled at intervals of 23, 29, and 34 seconds respectively. The second and third bells tolled 39 and 40 seconds respectively longer than the first. How many times did each bell toll if they all ceased in less than 20 minutes?

BOOKS RECEIVED

CHEMISTRY FOR THE NEW AGE, by Robert H. Carleton, Executive Secretary, National Science Teachers Association, Washington, D. C., in Consultation with Floyd F. Carpenter, Principal of Stivers High School, Dayton, Ohio. Cloth. Pages xiv+688. 15×22 cm. 1949. J. B. Lippincott Company, 333 West Lake Street. Chicago 6, Ill. Price $3.20.

THE SCIENCE OF CHEMISTRY, by George W. Watt, Professor of Chemistry, The University of Texas, and Lewis F. Hatch, Associate Professor of Chemistry, The University of Texas. Cloth. Pages viii+567. 15×23 cm. 1949. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. Price $4.50.

MATHEMATICS REVIEW EXERCISES, by David P. Smith, Jr., A.B., and Leslie T. Fagan, M.A., both Masters in Mathematics in the Lawrenceville School, New Jersey. Cloth. Pages vii+280. 15×23 cm. 1940. Ginn and Company, Statler Building, Boston 17, Mass. Price $2.00.

BIOLOGY IN DAILY LIFE, by Francis D. Curtis, Head of the Department of Science, University High School, Ann Arbor, Michigan, and Professor of the Teaching of Science, University of Michigan; and John Urban, Professor of Science, New York State College for Teachers at Buffalo. Cloth. Pages xv+608. 17.5×24 cm. 1949. Ginn and Company, Statler Building, Boston 17, Mass. Price $3.60.

FIELDBOOK OF NATURAL HISTORY, by E. Laurence Palmer, Professor of Nature and Science Education, Cornell University, and Director of Nature Education, Nature Magazine. Cloth. Pages x+664. 15×23 cm. 1949. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. Price $5.00.

GEOMETRY, A FIRST COURSE, by Paul L. Trump, University of Wisconsin, Madison, Wisconsin. Cloth. Pages xvi+496. 13.5×21.5 cm. 1949. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y.

INTRODUCTION TO ANALYTIC GEOMETRY AND THE CALCULUS, by H. M. Dadourian, Seabury Professor of Mathematics and Natural Philosophy, Trinity College. Cloth. Pages x+246. 13.5×20.5 cm. 1949. The Ronald Press Company, 15 East 26th Street, New York 10, N. Y. Price $3.25.

ANALYTIC GEOMETRY, by Alfred L. Nelson, Professor of Mathematics; Karl W. Folley, Professor of Mathematics; and William M. Borgman, Assistant Dean of Administration, all of Wayne University. Cloth. Pages viii+215. 13.5 ×20.5 cm. 1949. The Ronald Press Company, 15 East 26th Street, New York 10, N. Y. Price $3.00.

ELEMENTARY CALCULUS AND COORDINATE GEOMETRY, by C. G. Nobbs, Second Master, City of London School. Part II. Cloth. 399 pages. 13.5×21.5 cm. 1948. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. Price $4.50.

ULTRASONICS, by Benson Carlin, Hillyer Instrument Company, Inc., formerly with Sperry Products. Cloth. Pages xi+270. 15×22.5 cm. 1949. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. Price $5.00.

COLLEGE ALGEBRA, by Lewis M. Reagan, University of Wichita; Ellis R. Ott, Rutgers University; and Daniel T. Sigley, The Johns Hopkins University. Revised Edition. Cloth. Pages xiii+447. 13.5×21 cm. 1948. Rinehart and Company, Inc., 232 Madison Avenue, New York 16, N. Y. Price $4.00.

FUNDAMENTALS OF SYMBOLIC LOGIC, by Alice Ambrose, and Morris Lazerowitz, Associate Professors of Philosophy, Smith College. Cloth. Pages ix+310. 13.5×21 cm. 1948. Rinehart and Company, Inc., 232 Madison Avenue, New York 16, N. Y. Price $5.00.

RINEHART MATHEMATICAL TABLES, Compiles by Harold D. Larsen, Professor of Mathematics, Albion College, Albion, Michigan. Alternate Edition. Cloth. Pages viii+160. 14×21 cm. 1948. Rinehart and Company, Inc., 232 Madison Avenue, New York 16, N. Y. Price $1.00.

GENERAL CHEMISTRY, by A. W. Laubengayer, Professor of Chemistry, Cornell University, Ithaca, New York. Cloth. Pages xiv+528. 15×23 cm. 1949. Rinehart and Company, Inc., 232 Madison Avenue, New York 16, N. Y.

A SHORT COURSE IN DIFFERENTIAL EQUATIONS, by Earl D. Rainville, Associate Professor of Mathematics, The University of Michigan, Ann Arbor, Michigan. Cloth. Pages ix+210. 13×20.5 cm. 1949. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.00.

RURAL ARITHMETIC, by Orville L. Young, Ph.D., Associate Professor of Agricultural Education, Illinois State Normal University, Normal, Illinois. Cloth. Pages xii+303. 13×20.5 cm. 1949. The Bruce Publishing Company, 540 N. Milwaukee Street, Milwaukee 1, Wis. Price $1.96.

ANALYTIC GEOMETRY AND CALCULUS: A UNIFIED TREATMENT, by Frederic H. Miller, Professor of Mathematics, The Cooper Union School of Engineering. Cloth. Pages xii+658. 13.5×21 cm. 1949. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. Price $5.00.

A SHORT COURSE IN DIFFERENTIAL EQUATIONS, by Earl D. Rainville, Associate Professor of Mathematics, The University of Michigan. Cloth. Pages ix +210. 13×20.5 cm. 1949. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price \$3.00.

ANALYTIC GEOMETRY, by John J. Corliss, Irwin K. Feinstein, University of Illinois, Chicago Undergraduate Division, and Howard S. Levin, The Glen L. Martin Company. Cloth. Pages xiv +370. 13.5×21 cm. 1949. Harper and Brothers, 49 East 33rd Street, New York 16, N. Y. Price \$3.25.

OUR TEACHERS MOLD OUR NATION'S FUTURE, by Geraldine Saltzberg, Chairman, English Department, James Monroe High School, New York City. Cloth. Pages xv +189. 13.5×21 cm. 1949. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price \$2.25.

DIFFERENTIAL EQUATIONS, by Harry W. Raddick, Professor of Mathematics, New York University (University Heights). Second Edition. Cloth. Pages x +288. 13.5×21.5 cm. 1949. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. Price \$3.00.

HIGHER ALGEBRA FOR THE UNDERGRADUATE, by Marie J. Weiss, Professor of Mathematics, Newcomb College, Tulane University. Cloth. Pages viii +165. 13.5×21.5 cm. 1949. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. Price \$3.75

COLLEGE ALGEBRA, by Joseph B. Rosenbach, Professor of Mathematics and Assistant Head of the Department, and Edwin A. Whitman, Associate Profesor of Mathematics, Carnegie Institute of Technology. Third Edition. Cloth. Pages x +523 +xlii. 13.5×20.5 cm. 1949. Ginn and Company, Statler Building, Boston 17, Mass. Price \$3.00.

CALCULUS, by Lyman M. Kells, Professor of Mathematics, United States Naval Academy. Second Edition. Cloth. Pages xii +508. 15×23 cm. 1949. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price \$4.00.

BASIC COLLEGE PHYSICS, by Henry A. Perkins, Sc.D., Professor of Physics, Emeritus, Trinity College. Cloth. Pages ix +605. 14.5×23 cm. 1949. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price \$4.75.

SUGAR: ITS PRODUCTION, TECHNOLOGY, AND USES, by Andrew Van Hook, Professor of Chemistry, College of the Holy Cross. Cloth. Pages ix +155, 14×21.5 cm. 1949. The Ronald Press Company, 15 E. 26th Street, New York 10, N. Y. Price \$3.00.

INTRODUCTION TO COLLEGE GEOLOGY, by Chauncey D. Holmes, Associate Professor of Geology. Cloth. Pages xxi +429. 14×21 cm. 1949. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price \$4.00.

JAMES WATT AND THE HISTORY OF STEAM POWER, by Ivor B. Hart, Author of The Mechanical Investigations of Leonardo da Vinci, Makers of Science, The Great Physicists, The Great Engineers, etc. Cloth. Pages xii +250. 13.5×21 cm. 1949. Henry Schuman, Inc., 20 East 70th Street, New York 21, N. Y. Price \$4.00.

RADIO AND TELEVISION MATHEMATICS: A HANDBOOK OF PROBLEMS AND SOLUTIONS, by Bernard Fischer, Ph.D., Vice President in Charge of Training, American Television Laboratories of California. Cloth. Pages xviii +484. 13×20 cm. 1949. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price \$6.00.

BIOMETRICAL GENETICS: THE STUDY OF CONTINUOUS VARIATION, by K. Mather, D.Sc., Ph.D., John Innes Horticultural Institution, London. Cloth. Pages

ix+158. 14×21.5 cm. 1949. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $3.50.

INTRODUCTION TO THE THEORY OF FOURIER'S SERIES AND INTEGRALS, by H. S. Carslaw, Sc.D., LL.D., F.R.S.E., Professor of Mathematics in the University of Sydney. Third Edition, Revised and Enlarged. Cloth. Pages xiii+368. 13.5 ×20.5 cm. 1930. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $3.95.

AN INTRODUCTION TO COLLEGE GEOMETRY, by E. H. Taylor, Ph.D., Professor of Mathematics, Emeritus, Eastern Illinois State College, and G. C. Bartoo, A.M., Professor of Mathematics, Emeritus, Western Michigan College of Education. Cloth. Pages vii+143. 13×20.5 cm. 1949. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.15.

PHYSICS, THE STORY OF ENERGY, by H. Emmett Brown, Professor of Science, Head of Science Department, New York State College for Teachers, Buffalo, New York, and Edward C. Schwachtgen, Head of Department of Physical Science, Washburne Trade School, Chicago; Instructor of Physics and Radio, Chicago Evening High Schools. Cloth. Pages xi+593. 16×23.5 cm. 1949. D. C Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $3.20.

GENERAL CHEMISTRY, by Harry N. Holmes, Oberlin College. Fifth Edition. Cloth. Pages viii+708. 14×21 cm. 1949. Macmillan Company, 60 Fifth Avenue, New York 11, N. Y.

LIVING MATHEMATICS, by Ralph S. Underwood, Professor of Mathematics, Texas Technological College, and Fred W. Sparks, Professor of Mathematics, Texas Technological College. Second Edition. Cloth. Pages x+374. 15×22.5 cm. 1949. McGraw-Hill Book Company, Inc., 330 W. 42nd Street, New York 18, N. Y. Price $3.00.

MODERN-SCHOOL SOLID GEOMETRY, by Rolland R. Smith, Coordinator of Mathematics, Public Schools, Springfield, Massachusetts, and John R. Clark, Professor of Education, Teachers College, Columbia University. New Edition. Cloth. Pages viii+256. 13×20.5 cm. 1949. World Book Company, Yonkers 5, New York. Price $1.76.

CALCULUS, by Lloyd L. Smail, Ph.D., Professor of Mathematics at Lehigh University. Cloth. Pages xiii+592. 14×21.5 cm. 1949. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. Price $4.50.

COMMERCIAL ALGEBRA, by Clifford Bell, University of California, Los Angeles, and L. J. Adams, Santa Monica City College. Cloth. Pages vii+304. 13.5×21 cm. 1949. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. Price $2.75.

MATHEMATICS OF FINANCE, by Clifford Bell, University of California, Los Angeles, and L. J. Adams, Santa Monica City College. Cloth. Pages vii+366. 13.5×21 cm. 1949. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. Price $2.75.

MODERN PHYSICS, by Charles E. Dull, Late Head of Science Department, West Side High School, Supervisor of Science for Junior and Senior High Schools, Newark, New Jersey; H. Clark Metcalfe, Science Department, Brentwood High School, Pittsburgh, Pennsylvania; and William O. Brooks, Science Department, Technical High School, Springfield, Massachusetts. Cloth. Pages x+601+xxvi. 15×23.5 cm. 1949. Henry Holt and Company, Inc., 257 Fourth Avenue, New York 10, N. Y.

HOW TO KNOW THE IMMATURE INSECTS, by H. F. Chu, Ph.D., Zoologist, In-

stitute of Zoology, National Academy of Peiping, Peiping, China. 1946–47 Visiting Professor, Iowa Wesleyan College. 240 pages. 14×21.5 cm. 1949. Wm. C. Brown Company, Dubuque, Iowa. Price: Spiral Binding $2.00; Cloth Binding $3.00.

ANIMAL ENCYCLOPAEDIA MAMMALS, by Leo Wender. Cloth. 266 pages. 13 ×21.5 cm. 1949. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. Price $4.50.

PLANE AND SPHERICAL TRIGONOMETRY WITH TABLES, by J. Shibli, The Pennsylvania State College. Third Edition. Cloth. Pages xii +262 +94. 1949. Ginn and Company, Statler Building, Boston 17, Mass. Price $3.00.

THE CONCEPTS OF THE CALCULUS. A CRITICAL AND HISTORICAL DISCUSSION OF THE DERIVATIVE AND THE INTEGRAL, by Carl B. Boyer, Associate Professor of Mathematics, Brooklyn College. Cloth. Pages 346. 15×23 cm. 1949. Hafner Publishing Company, Inc., 31 East 10th Street, New York 3, N. Y. Price $5.50.

MATHEMATICAL FOUNDATIONS OF STATISTICAL MECHANICS, by A. I. Khinchin. Translated from the Russian by G. Gamow. Cloth. Pages viii +179. 12.5×19 cm. 1949. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $2.95.

ANALYTIC GEOMETRY, by W. A. Wilson, Late Professor of Mathematics, Yale University, and J. I. Tracey, Associate Professor of Mathematics, Yale University. Third Edition. Cloth. Pages x +318. 14.5×22 cm. 1949. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $2.75.

YOU CAN'T WIN: FACTS AND FALLACIES ABOUT GAMBLING, by Ernest E. Blanche, Ph.D., Author of Off to the Races, The Mathematics of Gambling. Cloth. 155 pages. 12.5×19.5 cm. 1949. Public Affairs Press, 2153 Florida Avenue, Washington 8, D. C. Price $2.00.

ALCOHOL AND HUMAN AFFAIRS, by Willard B. Spalding, Dean of the College of Education, University of Illinois, and John R. Montague, M.D., Vice Chairman, Educational Advisory Committee to the Oregon State Liquor Commission; Medical Director, Raleigh Hills Sanitarium for the Treatment of Alcoholism. Cloth. Pages vii +248. 13×19 cm. 1949. World Book Company, Yonkers-on-Hudson, New York. Price $1.64.

ELEMENTARY MATHEMATICS FROM AN ADVANCED STANDPOINT GEOMETRY, Volume II, by Fleix Klein. Translated from the Third German Edition by E. R. Hedrick, Vice President and Provost, The University of California, and C. A. Noble, Professor of Mathematics, Emeritus, The University of California. Cloth. Pages ix +214. 14×21.5 cm. 1939. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $2.95.

HYDROLOGY. PHYSICS OF THE EARTH—IX, Edited by Oscar E. Meinzer. Published under the Auspices of the National Research Council. Cloth. Pages xi +712. 15×23.5 cm. 1942. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $4.95.

NUCLEAR RADII OF THE SUN AND PLANETS AND THE THICKNESS OF THEIR ATMOSPHERES, by Louis William Flack, Sr. Book. 8 pages. 20×28 cm. 1949. Louis William Flack, Sr., 3215 E. 7th Street, Kansas City 1, Mo.

OUR SUN, by Donald H. Menzl, Ph.D., Harvard College Observatory. Cloth. Pages viii +326. 14.5×21.5 cm. 1949. The Blakiston Company, Philadelphia 5, Pa. Price $4.50.

BOOK REVIEWS

My Book about the American Continents, by Pearl H. Middlebrook. Paper. Pages 158. 20.7×28 cm. 125 figures, maps, pictures and graphs. 1948. Silver Burdett Company, New York, Chicago and San Francisco.

This most excellent work book is based on the Fifth Grade text, *The American Continents*, by Barrows, Parker and Sorenson. While its activities follow the textbook, they supplement the text and afford opportunity to use pictures, maps and graphs in expressing ideas and in securing data for geographic thinking.

The book provides a variety of materials and guidance in their use. It develops the skills needed in interpreting maps and pictures, and makes it possible for the child to use his knowledge in new situations.

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Teachers' Guide and Testbook, by Beatrice Collins. Paper. Pages 160. 20.7×27 cm. 1948. Silver Burdett Company, New York, Chicago and San Francisco.

This book accompanies *The American Continents* by Barrows, Parker and Sorensen. As its title suggests, it has been prepared for teachers. It is a highly useful aid and guide. It provides a time schedule for the successful mastery of the text; lists the specific understandings to be developed in each chapter; gives directions for reading, map study, picture study and the like; provides answers to the textbook exercises; furnishes supplemental reading lists and lists of songs; provides a testing program. The tests are excellent and may be reproduced in any way the teacher deems best.

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A World View, by Clarence Woodrow Sorensen, Department of Geography, Illinois State Normal University. Cloth. Pages v+410. 21×27.5 cm. 344 figures,—maps, sketches, photographs. Atlas,—additional maps. 1949. Silver Burdett Company, New York, Chicago and San Francisco.

The pupil who has become acquainted with the world through the study of the first three texts in the series to which *A World View* belongs, will find this new Junior High School text most stimulating and interesting. The book is well named, as it affords a world view and leads to an understanding of the interdependence of nations.

The text is well illustrated with black and white and colored photographs. It is well supplied with a variety of maps, graphs and statistical data. The Atlas has an unusually fine set of physical-political maps of the continents, as well as world maps (interrupted projection) showing distribution of world population, rainfall, temperature, etc.

The book is simply written. Of necessity, the handling of many countries is brief. The interdependence of people within nations and the interdependence of nations is well developed. The book provides a world view and upon the world understanding it develops can be fitted the world political and economic problems which are of major concern in later and more advanced grades.

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First Year College Mathematics with Applications, by Paul H. Daus, *Professor of Mathematics, University of California, Los Angeles;* and William M. Whyburn, *Professor of Mathematics, University of North Carolina, Chapel Hill.* Cloth. Pages xiii+239. 13.5×21 cm. 1949. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $5.00.

A text written for use particularly in engineering and technical schools. The content is intended to provide a strong background for work in calculus, to integrate the materials of college algebra, analytic trigonometry and geometry, and to illustrate all principles by applications from science and engineering so as to make the course independent of future use.

It is assumed that the student will have completed work including computational trigonometry. Lesson outlines are given for a one year ninety hour course, or a one semester course of seventy-five hours. Both oral and written exercises are provided, with answers in the back of the text for the latter. Review and supplementary topics are given at intervals throughout the various chapters. These include an introduction review, linear functions, single variable quadratic equations, the parabola, central conics, general conic relations, algebraic functions, and algebraic and analytic geometry treatments of trigonometric functions. Tables are given for powers and roots, four place logarithm and natural trigonometric functions, five place radian trigonometric functions, natural logarithms, and exponential and hyperbolic functions. All but the first of these are from the Macmillan Tables.

The development of a strong background for work in calculus depends upon the study of necessary materials, and student success in gaining sufficient ability and understanding of these materials. This text covers more than the topical content generally agreed to constitute such foundational work. In particular, curve sketching is well presented and used at various points throughout the text. Problems demand a high degree of manipulative skill even though little provision is made for drill type procedure. Omitted topics such as determinents, series, fractions, combinations, permutations, and probability certainly either do not measurably affect later work in calculus, or may be given more effectively when they are needed. This places the question of background development upon student success, with the assumption of the authors seeming to be that this will be improved by the study of integrated and applied materials.

The integration of materials is handled carefully and effectively. As examples, linear algebraic functions are related to the line properties of analytic geometry, algebraic and graphic quadratic system solutions with the study of central conics, and both algebraic and analytic geometry treatments of trigonometric functions are made utilizing curve sketching, complex numbers, polar and parametric equations. Extensive applications high light this integration, such as loaded beams, electric networks, engineering material configurations, harmonic functions, and alignment charts. Statistical method for handling empirical data are also developed to considerable degree.

Throughout the text the pure mathematical development is presented by carefully worded and concise statements followed by illustrative examples, whereas much greater exposition is made concerning applicative materials. These demand a fairly high degree of student understanding of scientific and engineering vocabulary. In the chapter on the parabola twelve pages are devoted to the development of standard forms and properties, with thirty-two pages of applicative materials. This comparison is by no means a common ratio throughout the text, but does serve to illustrate the general tone of emphasis. This presumes a quick grasp of mathematical relations, to be followed by intensive use of of these principles. It would seem then that successful use of this text would be confined to a much better than average group of students.

For the purposes as stated by the authors this seems to be an excellent text for use with competent and capable instruction and students having a strong degree of preparation. In addition it would well merit examination by all mathematics teachers as a rich source of applications and topics supplementary to usual course work in the first year of college mathematics.

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ELEMENTARY CALCULUS AND COORDINATE GEOMETRY, by C. G. Nobbs, *Second Master, City of London School*. Cloth. Two volumes: Part I, pages 255; Part II, pages 400. 15×22 cm. Part I: 1948; Part II: 1949. Oxford, at the Clarendon Press. Price: Part I, \$3.25; Part II, \$4.50.

This work, written "to round out the ordinary school mathematics course," emphasizes the contrast in the mathematics content in the English and American secondary school. The two volumes cover most of the material in our American college courses through calculus. The first volume is the more elementary, the author states that he is here concerned with ideas rather than techniques.

It is doubtful that this would be selected as a text in our schools, but the book is certainly a very valuable addition to a personal or school library. The first volume could be read by any honor high school student, the explanations in either volume could be of great value to a freshman or sophomore college student who is having difficulty in reading his own text. Of course certain terminology differs, for example the term *gradient* for slope or derivative; British monetary units, etc.

There is a wealth of problem material, ranging widely in difficulty, with answers provided to essentially all problems. There is much material which would be of value for mathematics club programs, for example: the game of battleships, lattices, Farey series many of the problems. The treatment in the second volume includes valuable methods or discussion not readily found in American texts, i.e., a fine diagrammatic outline of the possible loci for the general conic; an unusual method for expanding the binomial; an extensive discussion of approximation methods, particularly Newton's method.

Although there is relatively little new material in the sense of undiscovered results, one rarely encounters a work which seems to offer so much supplementary material. To a far greater degree than many texts, it would seem that this could be studied without a teacher—it might be of value for the secondary school teacher who wishes to refresh his knowledge of elementary college mathematics with which he has not had contact for several years.

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ANALYTIC GEOMETRY by Robin Robinson, *Professor of Mathematics, Dartmouth College*. First Edition. Cloth. Pages ix+147. 16×23.5 cm. 1949. McGraw-Hill Book Co., Inc., New York, N. Y. Price \$2.25.

The author of this text states that although analytic geometry is a necessary background for calculus, it should definitely be a course in geometry. Whether for this reason or in the interests of brevity the book departs in several ways from tradition. For example one does not find the usual formula for the area of a triangle with three points as vertices; the point of division formula does not appear until the middle of the text, and then in connection with harmonic separation (on the other hand in three dimensions the formula is introduced early); the two point form of the equation of a straight line is not given and the normal form is introduced only incidentally in connection with polar coordinates (there is no gain in brevity since the distance from a point to a line is developed by finding the foot of the perpendicular from the point to the line and then using the length formula); there is no treatment of directed distance from a line to a point and no discussion of the problem of obtaining equations of angle bisectors; parametric equations in two dimensions and conjugate hyperbolas are not discussed; although the concept is used, the term *system* (or *family*) of curves does not appear. One need not conclude from the omissions mentioned that all the "meat" of the course has been removed—the text treats in some detail such concepts as tangents, diameters, polars (including polars in space), tangent planes, rulings and circular sections on quadric surfaces.

The term *slope-angle* is used in place of *inclination*. Points often glossed over are frequently emphasized, for example in deriving the equation of the ellipse it is verified that squaring twice has not introduced points other than those on the locus. As examples of excellent treatment one might mention the careful distinction between "translation of axes" and the "replacement principle"; likewise the pointing out that for large values of x and y the term unity is negligible, hence the hyperbola is approximately represented by its asymptotes.

There are many exercises, a considerable portion of which involve development of geometric concepts new to the student, rather than mechanical substitution in a formula. There are no answers to the exercises.

Some instructors will prefer that radian measure be used in polar coordinate problems rather than degree measure alone; it might be asked why, since the normal form of the equation of a plane is used and from that the distance from a plane to a point is developed, the similar method was not used in two dimensions. As the author points out, the teacher must supplement the text—the curious student will ask why, for example, the space coordinate system used is called right handed; why a certain property is called the Boscovitch property (the text gives him no information). Some teachers will object to the innovation of introducing the equation of the tangent to a conic without proof, so that it may be used earlier. It may be debated whether the advantages gained offset the disadvantage of using mechanical substitution without understanding of the meanings involved.

Since opinions differ, some will be pleased, others displeased with the selection of topics and the methods of treatment. Before rejecting the text on the basis of omitted material one might well ask whether these portions are of real value, or have been retained purely from tradition.

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COMPARATIVE ANATOMY—AN INTRODUCTION TO THE VERTEBRATES, by Leverett A. Adams, *Professor of Zoology, University of Illinois;* and Samuel Eddy, *Professor of Zoology, University of Minnesota.* First edition. Cloth. Pages vii +520. 22.5×14.5 cm. 364 figs. 1949. John Wiley & Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. Price $5.00.

This is a new textbook for a college course in vertebrate zoology or comparative anatomy. It is divided into two parts: a review of the vertebrates, which consists of five chapters; and comparative anatomy of the vertebrates, in which a chapter is devoted to each of the systems. Because of its importance and greater study normally given it, three chapters are allotted to the skeletal system.

This is a complete revision of the senior author's "Introduction to the Vertebrates," first published in 1933. Emphasis, of course, is given the comparative antomy section. The authors have rewritten and reorganized the contents of the chapters to include recent findings. The material is organized in such a manner so that portions of chapters may be omitted if the instructor chooses to limit the amount studied. An extensive glossary of terms is at the back of the book.

The descriptions of the systems are not word weighty, and the drawings are clear in all the detail presented.

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BIOLOGY IN DAILY LIFE, by Francis D. Curtis, *Head of the Department of Science, University High School, Ann Arbor, Michigan, and Professor of the Teaching of Science, University of Michigan;* and John Urban, *Professor of Science, New York State College for Teachers at Buffalo.* First edition. Cloth. Pages xv +608. 24×17.5 cm. 422 illustrations. 1949. Ginn and Company, New York. Price $3.60.

Biology in Daily Life is a high school text designed to make the study of biology valuable and practical for all students who use it. There are eight units, with the following titles: problems and characteristics of living things, using our biological resources wisely, the world's food supply, food and life, the conquest of disease, behavior of living things, life continues from age to age, and kinds of life. Finally, there is a listing of the steps in the scientific method, a list of scientific attitudes, and a glossary of biological terms.

The book is organized to compensate for the various levels of ability among students. Sentences or paragraphs marked by an E (designating essential) permit the teacher and students to give those portions particular attention. Others are marked with a P to indicate biological principles, which can be applied generally to living things. At the end of each chapter there is a section entitled "Checking What You Know," consisting of multiple choice items for the student to answer. This is followed by a section entitled "Applying What You Know," containing thought questions which give the student an opportunity to use the knowledge he has acquired. Various other aids and stimulants to study are included. Their nature can be inferred from their titles: consumer biology, topics for individual study, experiments, why not become a naturalist, panel discussion, biology in the news, group investigations, community applications of biology, etc. And each chapter is concluded with a list of references and some with an additional list of books for leisure reading.

Biology in Daily Life includes information only recently incorporated in college texts, presented in such a fashion that it will not escape the comprehension of high school students. The text is well supplied with soft sepia-toned illustrations accompanied by questions to make the illustrations instructive.

One criticism which might be made of this text is the use of footnotes, excluding those of terminology pronunciation. Footnotes detract from the smooth flowing of text material. If the student uses them, he is interrupted. If he does not use them, they are a waste of space. Other than this the book is attractive and well written.

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INTRODUCTORY RADIO, THEORY AND SERVICING, by H. J. Hicks, M.S., *Radio and Science Instructor, Central High School, St. Louis, Missouri.* Cloth. Pages viii+393. 15×23 cm. 1949. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 18, N. Y. Price $3.20.

This is a well prepared and well written book. The material has been designed and organized to meet the radio teaching conditions as found in both the small and large high schools, with a real appreciation for the correct balance between radio shop practice and theory. While this book has been especially prepared for beginning classes in radio it will be of interest and real value to advanced radio students as well as being an excellent reference for artisans.

Theory is introduced at points where it can be immediately applied, and the book is the outcome of the author's belief that the radio science should be taught in an atmosphere filled with inspiration gained from every-day practical experiences. Difficult technical principles are explained in the language easily understood by a high school student, stressing the "learning-by-doing" approach. The diagrams and illustrations are excellent, and the questions and problems at the end of each chapter are thought-provoking. Necessary mathematics in as simple form as possible have been included. The book is of limited scope, but it should be remembered that it is written as a beginning text in radio.

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